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ADAPTION OF EVOLUTIONARY PROGRAMMING TO THE PREDICTION OF SOLAR FLARES

by Lawrence J. Fogel, Alvin J. Owens, and Michael J. Walsh

Prepared under Contract No. NAS 5-3907 *by*
GENERAL DYNAMICS/CONVAIR
San Diego, Calif.
for Goddard Space Flight Center

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Introduction

Evolutionary programming was originally devised as an alternative means for addressing some fundamental problems of artificial intelligence. In contrast to the bionic approach and heuristic programming, evolutionary programming does not attempt to replicate human performance, either biological or psychological. Instead, attention is focused upon the logic of Darwinian evolution as a means for generating a suitable logic for overcoming a wide variety of problems, as witness the survival of living organisms. The advent of high speed digital computers makes such replication practicable. Success in this regard may be expected to enhance our ability to analyze and understand the real-world.

Evolutionary programming was first restricted to problems of predicting binary environments. Demonstration of a considerable level of success in predicting certain difficult environments, such as the primeness of each successive positive integer, justified offering evolutionary programming as a means for the prediction and analysis of real-world time series. An unsolicited proposal by General Dynamics/Convair resulted in Contract NAS 5-3907 which was conducted under the cognizance of Dr. J. Lindsay and Mr. W. White of Goddard Space Flight Center/NASA. The assigned task included the following:

> Prepare various time series of known mathematical properties by utilizing the computer programs and mathematical analysis developed in the reported previous research.
>
> Offer these time series to the evolutionary prediction program as "environments" with the requirement that each next symbol be predicted prior to its "observation".

Employ various mutation noise random number sequences in order to furnish information from which it will be possible to provide explicit statements concerning the limits of predictability of these "control environments".

Submit time series data taken from actual measurements, provided by GSFC, to the evolutionary prediction program so that a careful report can be made of limits of their predictability.

Perform additional studies concerned with the precoding of such empirical data in order to facilitate application of the evolutionary technique.

Employ various methods which are intended to extract the underlying "signal" from the available data. These will include both deterministic and statistical properties.

Employ the services of a consultant to assist in making statistical comparisons of the results obtained with the evolutionary prediction technique to those obtained by conventional statistical means.

The intent of this contract was to develop a new technique for computation which would prove suitable for the analysis and prediction of sequences of measurements and for the identification of logical dependencies which might otherwise have gone unnoticed.

During the time of this contract research was also conducted by General Dynamics/Convair under contract to the Office of Naval Research. This report also includes some of the results of this contract in that these contributed to the clarity and completeness of presentation.

Discussion

Work under this contract began with the preparation of an evolutionary program having up to 8-symbols in either the input and output alphabets. This program was written in Fortran and symbolic language suitable for operation on the IBM-7094. Before describing this program in some detail it is well to review the fundamental concepts involved.

In essence the evolutionary program carries out a fast-time simulation of some of the logical aspects of natural evolution, at each point retaining the "organism" which appears most fit to cope with the given task. The organism takes the form of a mathematical entity which depicts a particular logic for transforming a sequence of input symbols into a sequence of output symbols. A "parent" organism is scored in terms of its ability to accomplish the desired "decision-making" on the basis of evidence at hand. This organism is mutated to yield an "offspring" which is then given the same task and scored in a similar manner. That organism which demonstrates the greatest ability to perform the required function is retained to serve as parent of a new offspring. Thus, non-regressive evolution proceeds to find better and better logical "programs" for attacking the problem at hand. At some point in real-time, or whenever a sufficiently good "program-machine" has been found, an actual decision is made. Such an evolutionary search for a most appropriate logic for the required decision-making is conducted each time new information is received.

To fulfill this simulation it is necessary to choose some mathematical representation for the organism. For the sake of simplicity the organism was allowed to take the form of a finite-state machine, that type of logical transduction which is specified in terms of a finite alphabet of possible input symbols, a finite alphabet of possible output symbols, and some finite number of possible different internal states. In order to identify such a machine each of these states must be described in terms of those symbols which would emerge from the machine when the machine is in that state and receives each each of the possible input symbols. To illustrate, a three-state machine is shown in Figure 1. The alphabet of input symbols is comprised of 0 and 1 while the output alphabet is comprised of α, β , and γ (as a matter of convention, input symbols are shown to the left of the virgule, while output symbols are shown to the right). As shown in the "state-diagram", an input of 1 to the machine while it is in state B will cause the machine to output an α, and change its internal state from B to C. On the other hand, the input symbol 0 causes an output symbol γ and the machine remains in state B. Thus, a sequence of input symbols is transformed into a sequence of output symbols as shown in Table 1, assuming that the machine is in state C when the first input is received. Obviously the output sequence would in general be different had the machine been in another "initial state". A finite-state machine, therefore, is completely specified by the state-diagram and an identification of its initial state (that state in which the machine is found when it receives the first symbol of the input sequence under consideration). It is assumed that the machine acts only when an input symbol is received and that this operation is without error and always completed before the next input symbol is received.

Table 1

Present State	C	B	C	A	A	B
Input Symbol	0	1	1	1	0	1
Next State	B	C	A	A	B	C
Output Symbol	β	α	γ	β	β	α

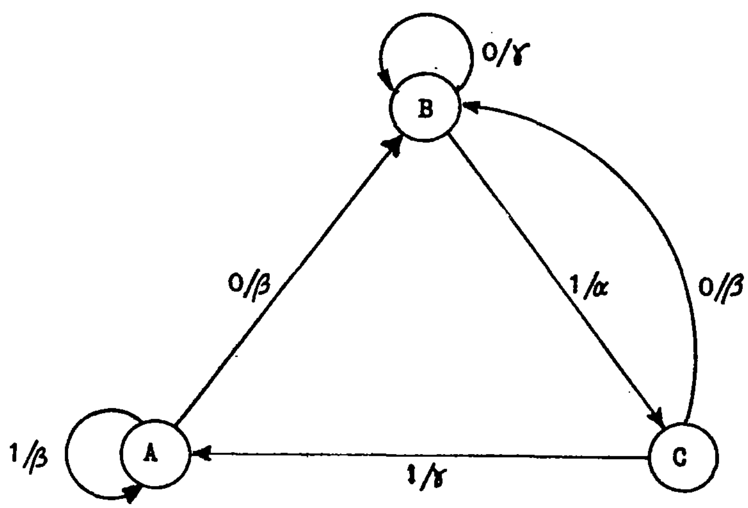


Figure 1

A Finite-State Machine

Now suppose that it is desired to predict each next value in a sequence of measurements. In other words, at each point in time the object is to devise an algorithm which will operate on the sequence of symbols thus far observed in order to produce an output symbol which is likely to agree with the next symbol to emerge from the sensed environment. To accomplish this, as arbitrary finite-state machine, M_0, is exposed to the sequence of symbols which have thus far emerged from

from the environment. As shown in Figure 2, each output symbol from the machine is compared with the next input symbol, the percent correct score being a measure of the "ability" of this machine to predict the already-experienced environment on the basis of each sequence of preceding symbols, reference Table 2.

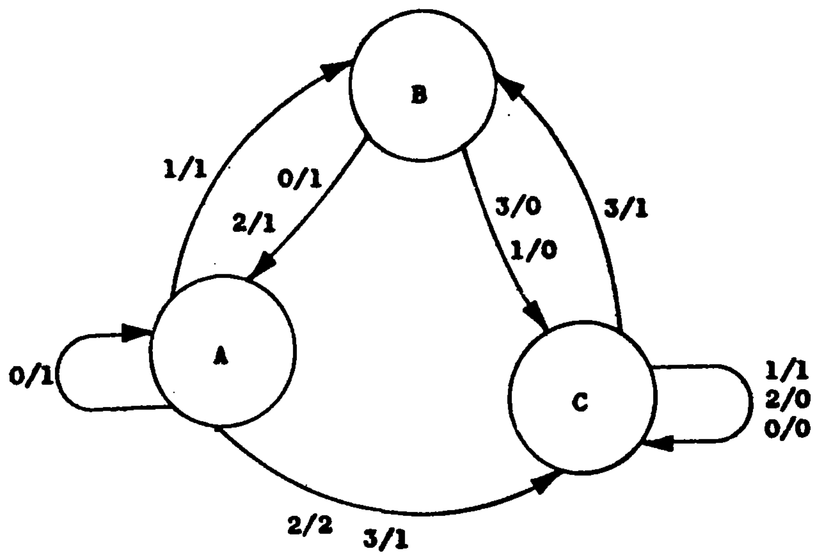


Figure 2
Machine M_0

Table 2

Present State	B	A	C	C	C	C	B	C	C	B	A	B
Input Symbol	2	2	1	0	1	3	3	0	3	0	1	2
Output Symbol		1	2	1	0	1	1	0	0	1	1	1
Error Cost		1	1	1	1	1	1	0	1	1	0	

The first row of this table indicates the sequence of states of the machine as it responds to the sequence of input symbols shown in the second row. State B was arbitrarily assumed to be the initial state. The most recently experienced symbol is shown to the immediate left of the vertical line, while the symbol to the immediate right of this line is the symbol to be predicted. The third row shows the sequence of symbols resulting from the transduction. Note that these symbols are shown each displaced one position to the right in order to facilitate comparison. The output symbol shown to the right of the vertical line is then this machine's prediction of the as yet unknown symbol.

The last row is generated by referencing the goal which is expressed in terms of an error-cost matrix, $A = a_{ij}$. For this example, $a_{ij} = 0$ if $i = j$, and $a_{ij} = 1$ if $i \neq j$. The "cost" of each transduction is found from the error-cost matrix; i referencing the machine output, and j referencing the next symbol of the input sequence. A measure, P_i, of the ability of the machine to predict each next symbol is calculated by summing the cost of the error and dividing this sum by the number of terms. Thus, the average cost of the errors of machine M_o is found to be

$$P_1 = \frac{\text{Sum of the Error Costs}}{\text{Number of Terms}} = \frac{8}{10} = 0.8$$

An "offspring" of this machine is then produced through mutation, that is, through a single modification of the "parent" machine in accordance with some mutation noise distribution. The mode of mutation is determined by the interval within which a number selected from a random number table lies. The intervals are chosen in accordance with a probability distribution over the permitted modes of mutation. Additional numbers are then selected in order to determine the specific details of the mutation. Thus the offspring is made to differ from its parent either by an output symbol, a state-transition, the number of states, or the initial state. In the case of a 2-symbol environment, a deterministic procedure can be used to replace this type of mutation. As each symbol from the environment is predicted on the basis of the preceding symbols, score is maintained of the relative frequency of success of each state-transition. A predictive-fit score of greater than 0.5 can then be ensured by the reversal of output symbols on those state-transitions which were "more often wrong than right". For example, the offspring of M_o, M_1, might appear as shown in Figure 3 where the two differing output symbols were set deterministically.

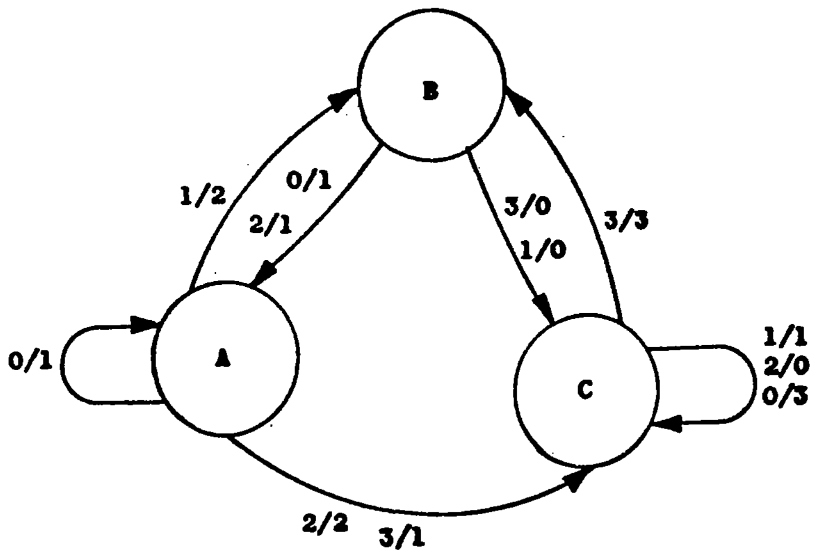


Figure 3. Machine M_1

Table 3 indicates the evaluation of the prediction capability of this machine over the same sequence of experience (this being termed the "recall").

Table 3

Present State	B	A	C	C	C	C	B	C	C	B	A	B
Input Symbol	2	2	1	0	1	3	3	0	3	0	1	2
Output Symbol		1	2	1	3	1	3	0	3	3	1	2
Error Cost		1	1	1	1	1	0	0	0	1	0	

The resulting average cost of the errors, P_1, is found to be 0.6. This offspring, therefore, demonstrates a superiority over its parent with respect to the given task, hence the original parent, M_o, is discarded and the offspring, M_1, is retained to serve as a parent. If machine M_1 had a predictive-error in excess of 0.8, it would have been discarded and the next evaluated offspring would be generated by mutating machine M_o.

At any point in time the remaining machine can be used for actual prediction. As seen in Table 2, the machine M_o had predicted that the next symbol to emerge from the environment would be a 1. This was incorrect. Table 3 indicates that the machine M_1 correctly predicted the next symbol to emerge from the environment. The risk associated with each prediction is roughly related to the average of the errors in "predicting" the past.

Machine M_1 is now used to parent succeeding offspring which are evaluated over the same recall. Thus the evolution continues in fast-time in preparation for an actual prediction. Such prediction may take place periodically, aperiodically, or upon request. It may be made whenever the average score in "predicting" the past has fallen below some specified value, when some prechosen number of offspring have been evaluated, or when an appropriate number of generations have occurred.

Of course, in general, the longer the time interval before each prediction, the greater the expectation of success. Similarly, the greater the speed of the computer facility (increase in the number of evaluated offspring) or the larger the available memory (increase of their permissible size) the greater the evolutionary prediction capability.

The process continues in this same manner. After each actual prediction the remaining machine serves to generate successive offspring. In essence, the evolutionary program provides two processes: there is the iterative mutation and selection of machine...a continual search for that decision-logic which would have been "best" had it been used in the past, and there is the read-out procedure which carries the remaining machine one step into the future to yield an actual prediction whenever such a prediction is required. If it should happen that some of the data from the environment are found to have been in error, it is only necessary to make the appropriate changes to correct the recall.

The goal, as referenced above, was expressed in terms of a particular set of costs associated with each of the possible correct and incorrect predictions of the next symbol to emerge from the environment. Note that the goal is not restricted to this set. For example, if the goal is to minimize the magnitude of the error of the prediction of each next symbol, the elements of the error-cost matrix would take the values $a_{ij} = (i - j)$. If it is desired to minimize the mean squared error of such prediction the elements would become $a_{ij} = (i - j)^2$. Of, if "a miss is as good as a mile", the error matrix should have equal non-zero off-diagonal terms and zero on the diagonal. In fact, if there is some greater worth associated with correctly predicting a certain symbol, this can be reflected in the cost matrix expression of the goal. For example, the goal expressed in Table 4 indicates that it is most important to properly predict the symbol 0, less important to correctly predict the symbol 1, and least important to correctly predict the symbol 2. At the same time, this goal expresses the fact that it is more desirable to err by predicting a 1 when the actual symbol is a 0 as opposed to predicting a 0 when the actual symbol is a 1. The relative worth of each possible outcome is expressed in such a statement of goal. And there is no additional cost in using such a complex goal. As each output symbol of the machine under evaluation is compared to the next symbol which actually emerged from the environment, reference is made to the given goal matrix. The sum of the associated costs provides the desired measure of the worth of the particular machine in terms of its ability to predict the sequence of symbols which have already been experienced. In general, then, the sequence of machines which will evolve will be a function of the goal. Further, there is no need to restrict the evolution to a single invariant goal. If the goal is changed as real-time experience proceeds, the sequence of retained machines will gradually reflect the change of goal. Of course, all that has been said concerning goals expressed in the form of penalty matrices applies equally well if the goal is expressed in the form of pay-off matrices.

Table 4

Actual \ Predicted	0	1	2
0	0	3	4
1	4	1	4
2	4	4	2

The evolutionary technique offers even greater versatility. For example, the desire to predict each <u>second</u> symbol in the future can be satisfied simply by scoring each offspring in terms of the correspondence between its output symbols and those symbols which emerge from the environment two symbols later. By the same token, appropriate scoring of the offspring permits the prediction of any particular future symbol, the average of some set of future symbols, or indeed, any well-defined function over the future.

Note that the evolutionary program as described above is suitable for the prediction of multivariate environments. The symbols of the sensed parameters of the environment may be combined into a new alphabet of symbols which describes the environment in an unambiguous manner. The resulting predictions are made in the same alphabet so that they may be interpreted in terms of separate predictions for each of the parameters. Of course, the number of symbols in such a languages rises rapidly as a function of the number of variables. However, in principle, there is no restriction.

But the purpose of evolutionary programming need not be restricted to prediction in any sense. For example, problems of interdiction may be addressed by comparing the input variable derived from the environment with another parameter of the environment which exists concurrently but can only be measured in retrospect. Here is the case of <u>diagnosis</u>. A number of parameters of the system under consideration are measured. The intent is to determine some other parameter which is not open to immediate measurement.

Note that in the case of interdiction the input and output languages of the evolving machines might be different. The input symbols may be individually associated with a set of possible stimuli; the output symbols with a set of alternative responses, and the goal is to optimize the performance with respect to some well-defined evaluation of the response. A distinction need no longer be made between prediction and the translation of the prediction into a response. The evolutionary program recommends each action in the light of its expected over-all worth with respect to the given goal.

In greatest generality, a goal is completely specified only if it includes a statement of the relative importance to the decision-maker of each point in future time and there is a complete penalty or payoff matrix expressed for each point in the future. To this point, it has been tacitly assumed that the importance function of the goal-seeking entity has been restricted to a single point in its future time. That is to say, if the concern is to predict the next symbol no importance is associated with the second symbol into the future, or for that matter with any other symbols farther into the future. In principle, it is possible to accept a more complex importance weighted expression of goal in that each offspring can be continued to yield its prediction over the indefinite future. Each of these can be translated into their relative worth. Once the actual environment becomes known, the evaluation of the logic is then found by combining these metrics by means of relative importance weighting. Note the importance of having available actual information concerning the environment. The prediction of sequences of symbols becomes of interest in situations wherein measurements of the environment can only be made at certain times. In its most general form evolutionary programming provides a means for seeking any well-define goal in a manner consistent with the allowable costs. Each offspring is evaluated in terms of its own total worth as an organism...the value of its sequence of actions in the light of previous circumstances is degraded by the cost of its very existence.

Efficiency of the simulated evolution can be improved in a number of ways. For example, any available information concerning the underlying logic of the environment can be used effectively by translating this logic into the form of a finite-state machine which would express a proper logic for the desired decision-making under such environment. This machine may then serve as the initial machine. If this "hint" is reasonably correct, the evolution may require fewer generations to attain the same score. If it is incorrect, this introduction of "false" information in no way precludes solution of the problem, although it may be expected to reduce the efficiency of the procedure.

By the same token, the evolutionary technique permits the examination of "tentative results" at any point during the process. If a sufficiently worthwhile design has not yet evolved, the evolution may be reinstituted using as the initial machine the previously resulting machine. In this manner the procedure may be extended until some suitable goal-directed complexity-costed "organism" has evolved or the available expenditure has been exhausted.

The maxim of parsimony may be directly incorporated into the evolutionary procedure by increasing the penalty of each machine by a measure of its complexity. The amount of this penalty may be influenced by the particular characteristics of the computer facility upon which the simulation is to be carried out. For example, the penalty for complexity might take the form of a constant times the number of states in the machine under evaluation. If the available memory is severely restricted, it might be desirable to use a constant multiplied by the square of the number of states or some other suitable expression. In any case, the procedure should restrict the offspring from growing to size beyond the available memory of a computer. In general, the cost for complexity may take any form so long as its value may be directly computed from the specification of the machine.

If the environment is primarily statistical in nature, it is reasonable to suspect that a smaller machine will provide a suitable representation for the decision logic. Such a machine may characterize conditional dependencies and because of its small size have considerable statistical validity associated with each transition. On the other hand, such a small machine might present an over simplified "view" of a complex deterministic environment. It becomes evident that the penalty for complexity provides a parameter which can be adjusted over a scale ranging from a purely statistical view to a purely deterministic view of the environment. The investigator can have the selection of this parameter at his disposal. However, if his decision is made on insufficient grounds, he may have imposed an unnecessary restriction on the evolutionary capability. If is, of course, possible to provide a procedure which will continually optimize the cost for complexity as the program is exercised on the environment. For example, this procedure might include the comparative evaluation of a pair of machines prior to the generation of each offspring. The "functional-fit" (average cost of the error) indicates which of these machines is most suitable. Reference to its penalty for complexity as compared with that of the other machine furnishes a basis for its suitable modification. Thus, as the evolutionary program proceeds, it should continually revise its "opinion" as to the nature of the environment, this ranging over a scale from purely statistical to purely deterministic.

Thus, at each point in time, the evolutionary technique provides a nonregressive search through a domain of finite-state machines under the constraints imposed by the available computation capability for that machine which is most likely to achieve the given goal. This search may be viewed as a selective random walk, a "hill climbing" procedure, in a hyperspace defined to include the finite-state machines and an additional coordinate on which is measured the average cost of the error score. The danger of becoming trapped on a secondary peak can be overcome by permitting multiple mutation, with the multiplicity being a function of the difference in the average error score of successive generations. Thus, as the search nears a peak greater and greater "attention" may be devoted to generating more radical offspring in the hope of striking a point which may lie higher on the slope of another peak.

In general, a suitable choice of the mutation noise can increase the efficiency of evolution. For example, an increase in the probability of adding a state generates a wider selection of larger machines which should benefit evolution against a complex environment. In fact, the probability distribution over the modes of mutation can be made to depend upon the evidence acquired within the evolutionary process itself. Thus, an experienced greater relative frequency of success for, say, changing the initial state might be made to increase the probability of this mode of mutation.

Evolutionary programming can be used even in the face of nonstationary environments because it provides an iterative search for a "best" logic. But selection of only the single best logic may be an overly-severe policy. Certainly all those offspring which demonstrate a significant "decision" capability in their evaluation over the recall characterize the environment in some meaningful manner. Why not mimic natural evolution and save the best machines at each point in time? In general, the "best" offspring is most likely to give rise to a superior offspring, thus it should receive most attention in terms of mutative reproduction. Lower-ranked offspring may be regarded as insurance against gross nonstationarity of the environment. The distribution of mutative effort may well be in proportion to the normalized evaluation scores. Evaluated offspring are inserted into the rank order table of retained offspring and a <u>generation</u> is said to occur whenever an offspring is found which has a score equal to or greater than the score of the best machine. All of the retained machines need not lie on the slopes of the peak which is identified by the best machine. Thus, saving the best few offspring may maintain a "cognizance" over several peaks, with the relative search effort being distributed in proportion to the expectation of significant new discoveries.

The recombination of individuals of opposite sex appears to benefit natural evolution. By analogy, why not retain worthwhile "traits" which have survived separate evaluation by combining the best surviving machines through some genetic rule; mutating the product to yield offspring? Note that there is no need to restrict this mating to the best two surviving "individuals". In fact the most obvious genetic rule, majority logic, only becomes meaningful with the combination of more than two machines.

It is always possible to draw a single state-diagram which expresses the majority logic of an array of finite-state machines. Each state of the majority logic machine is the composite of a state from each of the original machines. Thus the majority machine may have a number of states as great as the product of the number of states in the original machines. Each transition of the majority machine is described by that input symbol which caused the respective transition in the original machines, and by that output symbol which results from the majority element logic being applied to the output symbols from each of the original machines. To illustrate, Figure 4 indicates three original machines and their individual probabilities of success, p_i. The initial state of each machine is indicated by a short arrow pointing to that state. The output of these machines is to be combined through a majority logic element which weights the importance of each output by the demonstrated probability of success of that machine. The resulting majority machine is shown in Figure 5.

So long as there are only two original machines the weighted majority logic machine reduces to the better of these two machines. A more interesting situation occurs when there are three original machines, these being rank-ordered so that $p_1 \geq p_2 \geq p_3$. In the 2-symbol case, in view of the above described deterministic output symbol reversing technique, these scores cannot be less than 0.5; therefore, it will always be true that $p_1 \leq p_2 + p_3$. Because of this inequality the weighted majority machine reduces to the simple unweighted majority machine.

The score for the majority machine, p , can be expressed as the sum of the ways in which success can be attained, that is,

$$p_M = p_1 p_2 p_3 + p_1 p_2 q_3 + p_1 q_2 p_3 + q_1 p_2 p_3$$

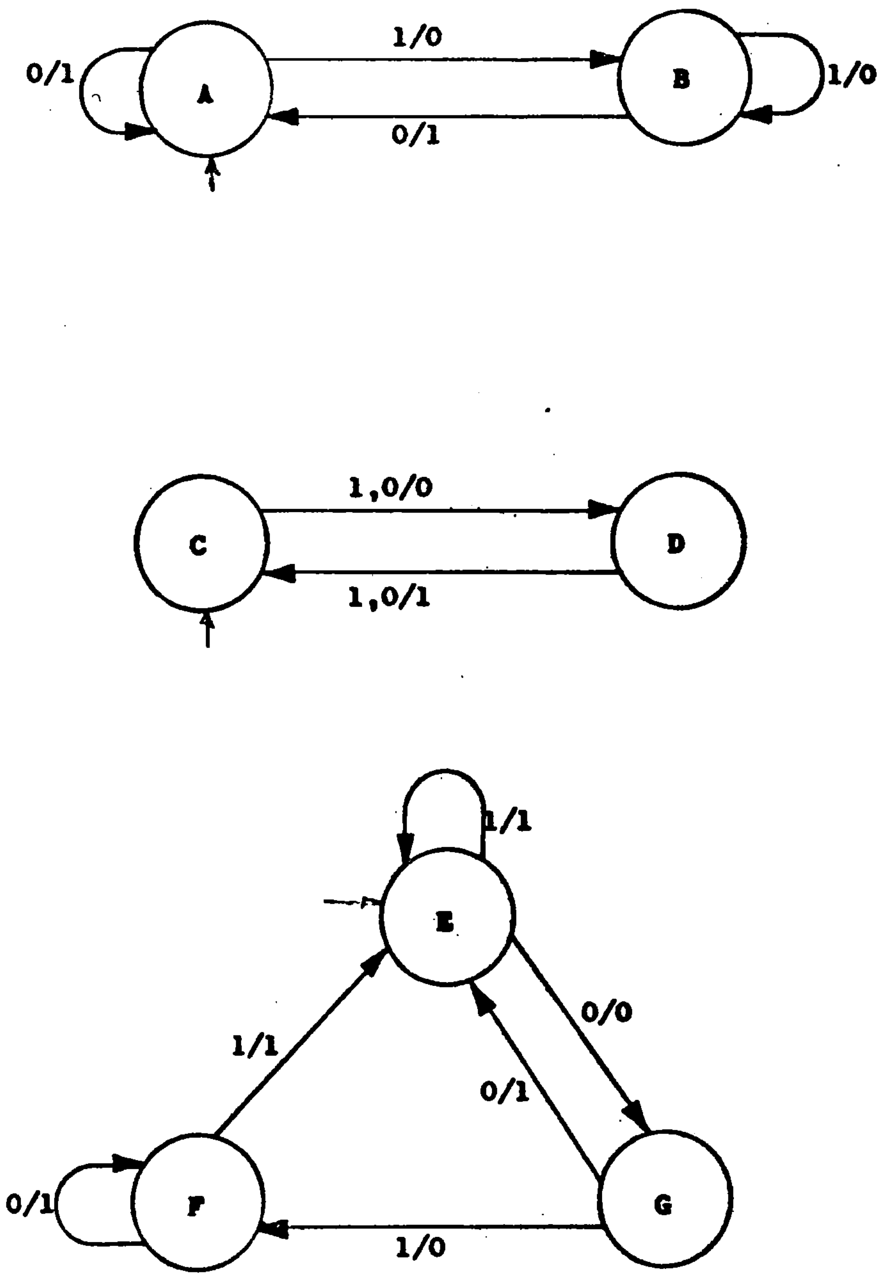


Figure 4 Individual machines to be combined

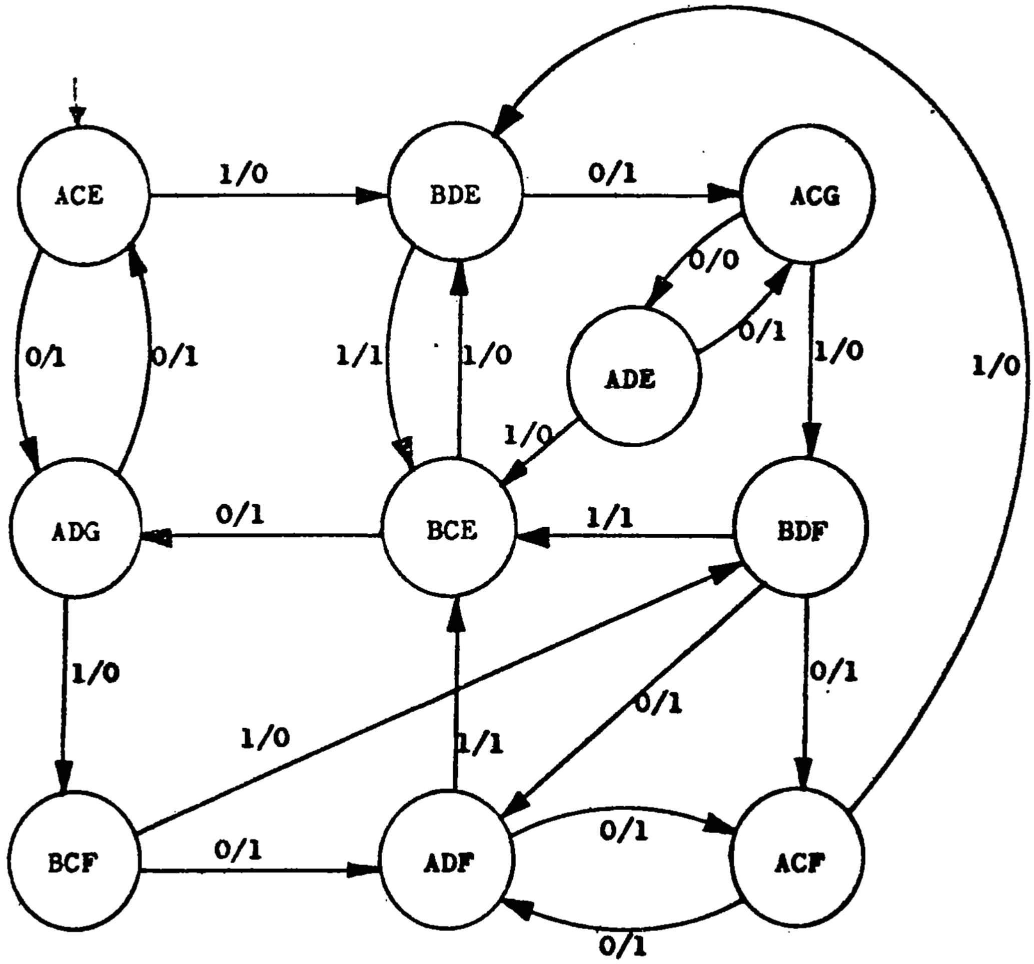


Figure 5 The resulting majority logic machine

where q_i is the probability of failure, so that $p_i + q_i = 1$. This expression can be simplified to

$$p_M = p_2\, p_3\, (1 - 2p_1) + p_1\, (p_2 + p_3)$$

Now the majority machine is only of value if $p_M \geq p_1$. That is if $p_M - p_1 \geq 0$. Substituting yields the inequality

$$p_2\, p_3\, (1 - 2p_1) + p_1\, (p_2 + p_3) - p_1 \geq 0$$

This relationship can be expressed in graphical form for specific values of p_1 and solved for maximum value of p_M. Figure 6 shows the regions in which pairs of values of p_2 and p_3 yield a $p_M \geq p_1$ for these values of p_1. Table 5 indicates the corresponding values for the maximum p_M.

Table 5

The Maximum p_M Related to Specific p_1

p_1	Max p_M
0.6	0.648
0.7	0.784
0.9	0.972
0.99	0.9997

This same procedure can be used for the evaluation of the benefit to be achieved through a majority logic element over any number of original machines; however, the required calculations rapidly become cumbersome. Further, for large numbers of original machines it becomes worthwhile to examine the behavior of the weighted majority logic machines as opposed to the simple majority logic machine. Clearly, this opens the door to many new possibilities. For example, it might prove fruitful to explore the combining of the best machines of several different

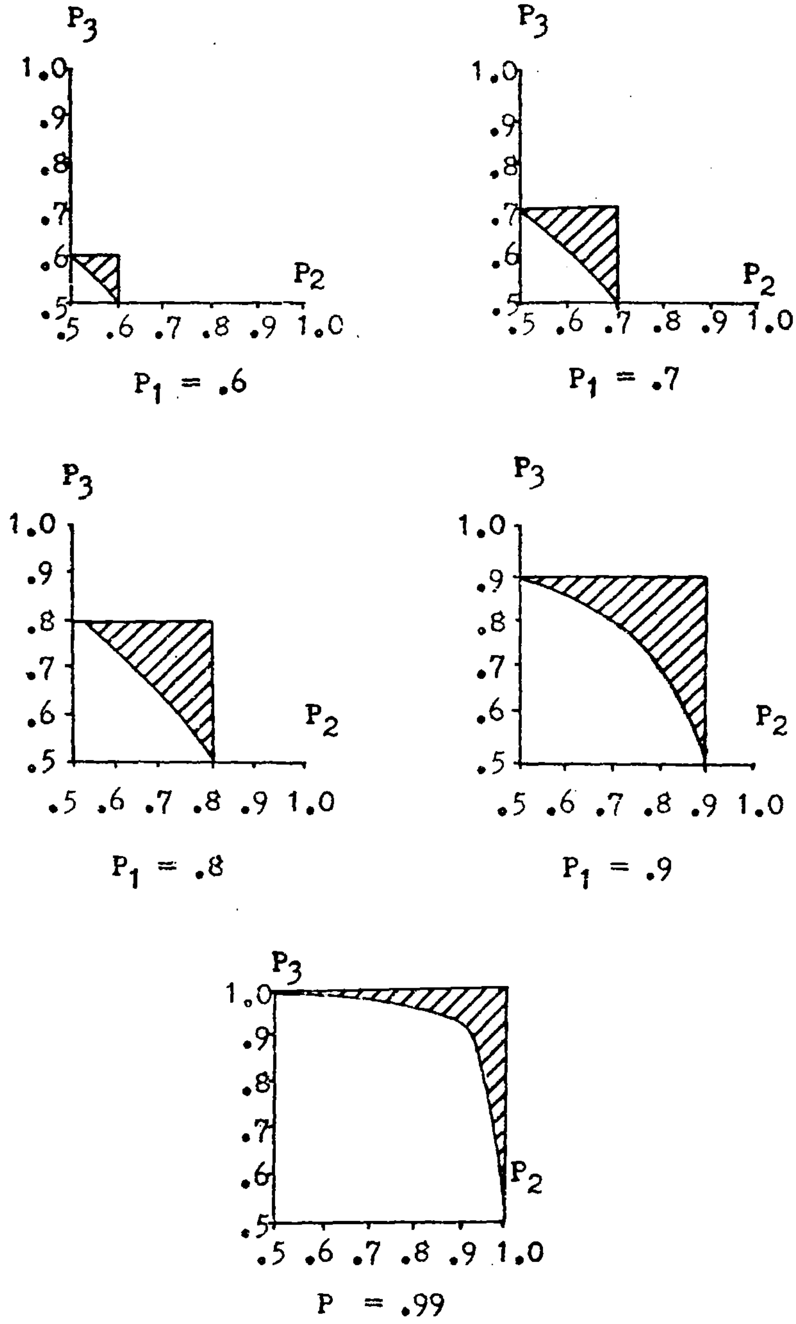


Figure 6. Majority Logic Improvement Domains

generations in the hope of finding a model of the models which had this far proven to be "most successful".

Thus far it has been tacitly assumed that each offspring is to be evaluated over the entire available experience. Obviously, this is not always the best policy. For example, if the environment changes its statistical properties it may be better to restrict the recall to some past sequence of symbols in order to enhance the probability of finding a most suitable decision logic for the present. In the face of complete ignorance concerning the logical construction of the environment, the choice of a particular length of recall might prove to be disastrous. It is, however, possible to use a procedure similar to that which was suggested above for determining successively more appropriate values for penalties for complexity. At each point in time, two or more machines are evolved over different recall lengths and their individual worth is compared resulting in a means for determining the next lengths of recall to be considered. So long as possible, the complete experience should be retained so that the recall length may be allowed to grow if this appears to be desirable. The problem becomes increasingly difficult with the restriction of the available memory to the point where the only alternative is "slide"; that is, keeping the recall length fixed.

All of these procedures which are intended to increase the efficiency of evolution offer a danger if the environment is interactive. An intelligent adversary might discover the specific procedure and use this knowledge to construct an obverse strategy.

Significant difficulties are encountered in attempts to express the principles of evolutionary programming in terms of conventional mathematics. The succession of finite-state machines which are considered is dependent upon the statistical process of mutation and upon the environment. Thus, any formal representation of the process would have to be restricted to a special class of environments. But the very point of evolutionary programming is its versatility. Here is a procedure which includes a random process in order to generate a never-ending sequence of alternative hypotheses concerning suitable logic and decision-making. However, information from this source is selected in the light of the given goal and the experienced environment.

Classical techniques for decision-making are based on the presumption that the environment is independent of the decision-maker and that the environment is stationary or can be viewed as stationary through a simple transformation. Neither of these classical assumptions is required for evolutionary programming. Its worth is achieved through the inheritance of logical properties and through the extensive search made possible by the advent of high speed digital computers.

The narrative description of the logical flow of an 8-symbol evolutionary program can better be followed by referring to Figures 7, 8, and 9. In order to properly identify each computer run the input data is first read-in and printed-out. The initial trial machine is then exercised over the recall in order that the experience gained can be utilized in deterministically setting the output for each state-input pair so that a minimum machine score over the recall is attained. This is done by determining that output of those required for each state-input pair which results in the smallest error sub-score for each transition. Error terms are determined by reference to the goal matrix. The output

LOGICAL FLOW OF AN EVOLUTIONARY PROGRAM

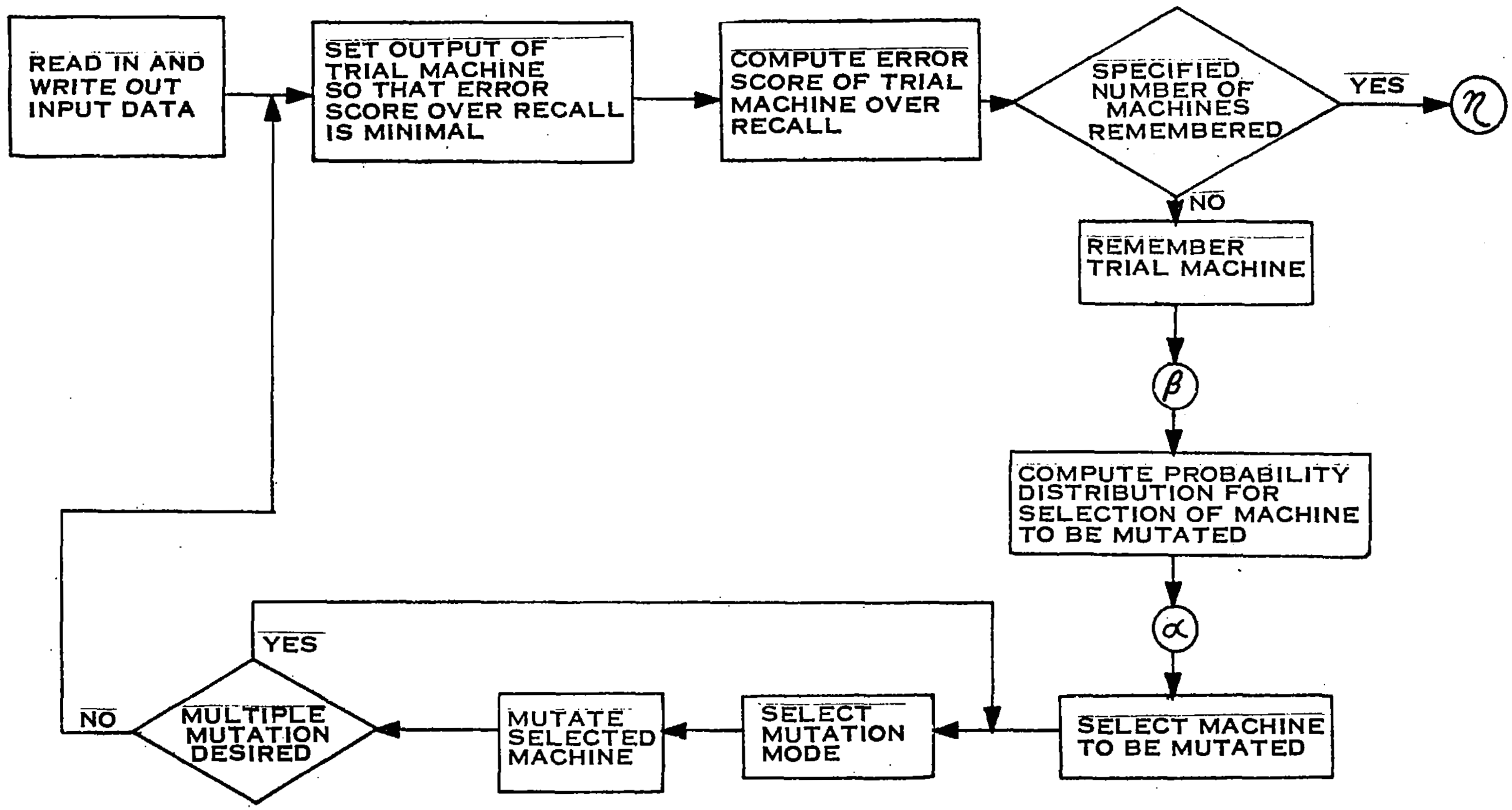


FIGURE NO. 7

LOGICAL FLOW OF AN EVOLUTIONARY PROGRAM (CONTINUED)

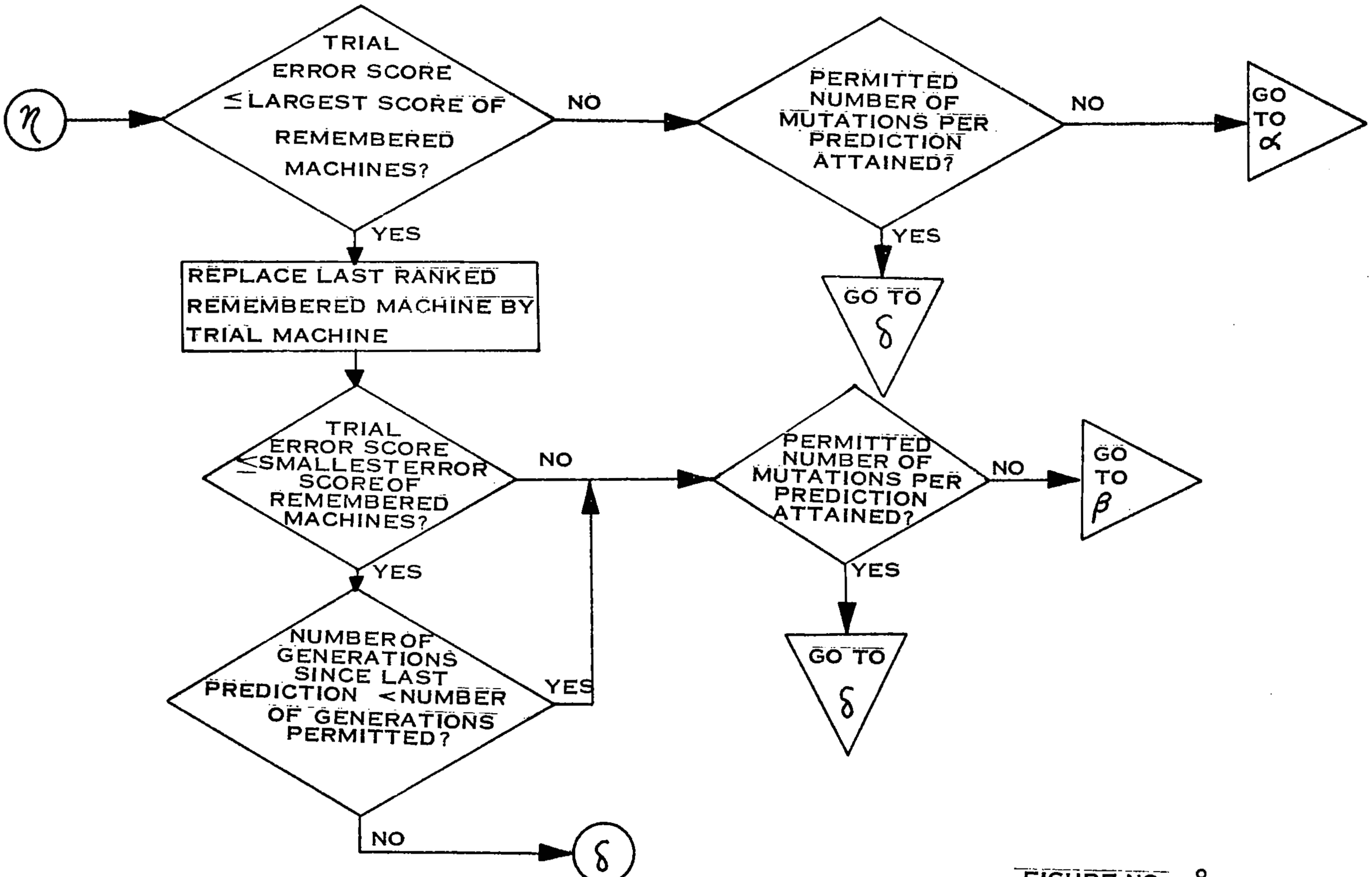


FIGURE NO. 8

LOGICAL FLOW OF AN EVOLUTIONARY PROGRAM (CONTINUED)

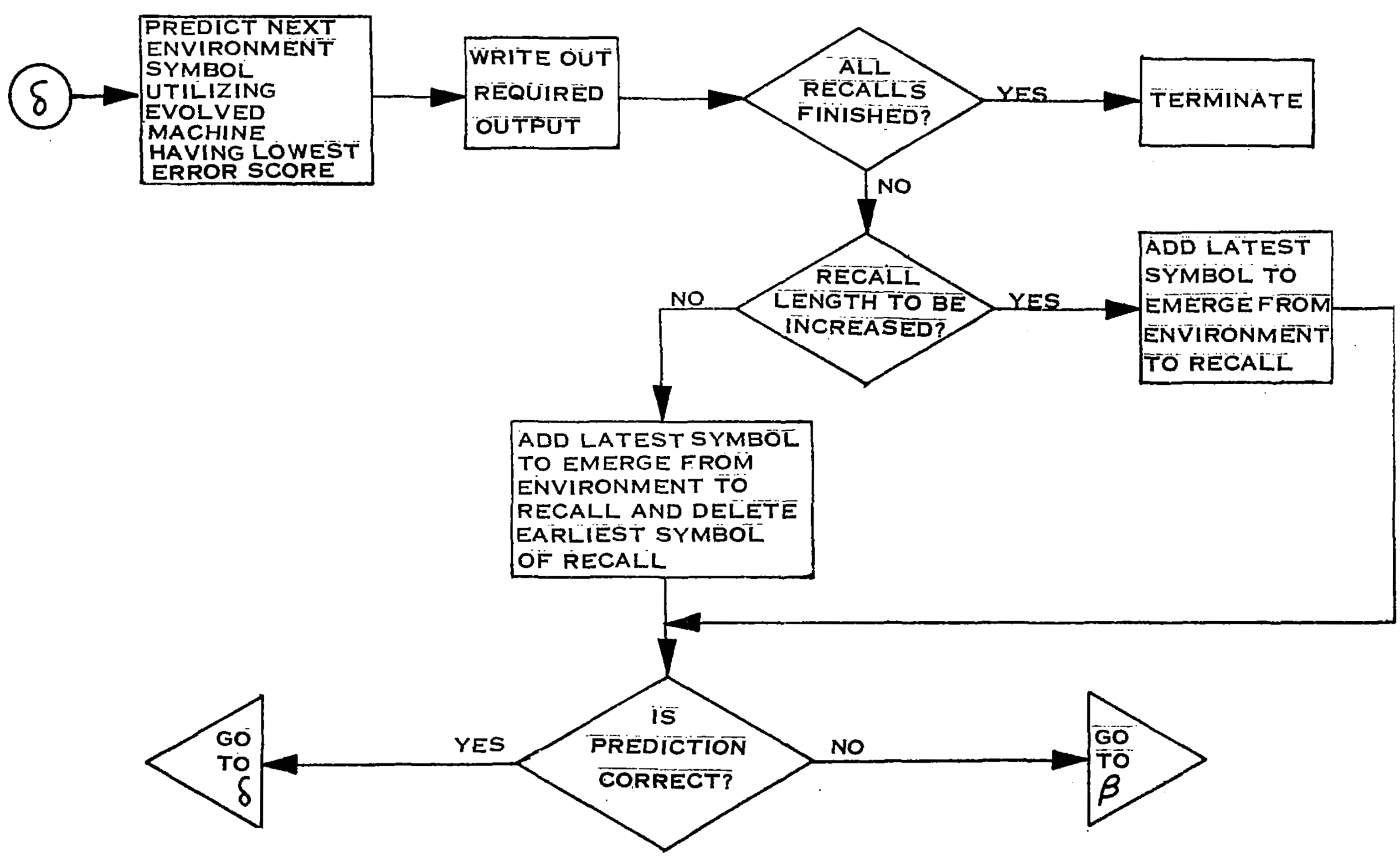


FIGURE NO. 9

for a state-input pair not exercised over the recall is set to that symbol which has so far been experienced most frequently. Next, the error score for the trial machine is computed over the recall by reference to the goal matrix. The penalty factor (if it is to be considered) is now added to the error score.

Next a test is made to determine whether or not the specified number of machines have been remembered. If not, the trial machine is stored in the computer memory.

A probability distribution by means of range intervals for selection of the next machine to be mutated is then computed in the following manner: the reciprocals of the error scores are summed. (The error score is understood to be the sum of the error sub-scores for each state-input pair plus the penalty factor, if applicable.) Each reciprocal is divided by this sum in order to obtain normalized scores. Range intervals are calculated as the cumulative sum of the normalized scores. The range interval associated with each stored machine is inversely proportional to its error score.

The machine to be mutated is determine by a random number R (where $0 \leq R < 1$) falling within the range interval associated with a stored machine. The selected machine is moved to the trial area.

Selection of a mutation type is determined by a random number falling within a mutation type distribution interval which was read in as input data. After the trial machine is mutated a test is made to determine whether or not an additional mutation is to occur. If so, mutation type selection and mutation occurs as described. If on the other hand, no further mutation is required, a transfer is made to that portion of the program wherein the output symbol for each state-input pair is set deterministically.

Attention is returned to the test which determines whether or not the specified number of machines has been stored. Assuming the required number of machines has been remembered, a comparison is made of the trial score and the largest score of the remembered machine scores. If the trial score is less than or equal to the largest score of the stored machines, that machine is replaced by the trial machine.

A further comparison is then made of the score of the trial machine and the smallest score of the remembered machines. If the trial score is less than or equal to the smallest score, a generation is said to have occurred. A comparison of the number of generations occurring since the last prediction is made with the maximum number of generations permitted per prediction (a number read in as input data). If the maximum permitted number of generations per prediction has been attained a prediction is required.

The last known symbol of the recall together with the final internal state of the trial machine having the lowest error score, a state-input pair, is used as input in stimulating the machine for an actual prediction of the next symbol.

A comparison of the predicted symbol with the next symbol to emerge from the environment as well as other pertinent information is then written as output.

The next test determines whether or not data processing is to continue. If not, the program terminates. However, if further prediction is required, a test is made to determine whether growth or slide is desired. Growth dictates adding the latest symbol to emerge from the environment to the recall with corresponding adjustment of the error scores of each remembered machine to include the error term would appear that this last increase in the noise level (from 39.67% in Experiment 4 to 43.8% in Experiment 5) resulted in significantly degraded prediction of the environment.

Figure 12 indicates the degree of correspondence between the sequence of predictions and the signal in these experiments. Note that after the first 76 predictions the signal was predicted in Experiment 5 as well as it was in Experiment 4 in spite of the fact that a larger percentage of the signal-symbols had been disturbed. This may be due to the fact that in the last experiment the symbols remained closer to the original signal. Such "consideration" for the magnitude of deviations is a result of using a distance-weighted error matrix, in this case the weighting being the magnitude of the symbol difference. In essence this choice converts the nominal scale of symbols to an ordinal scale.

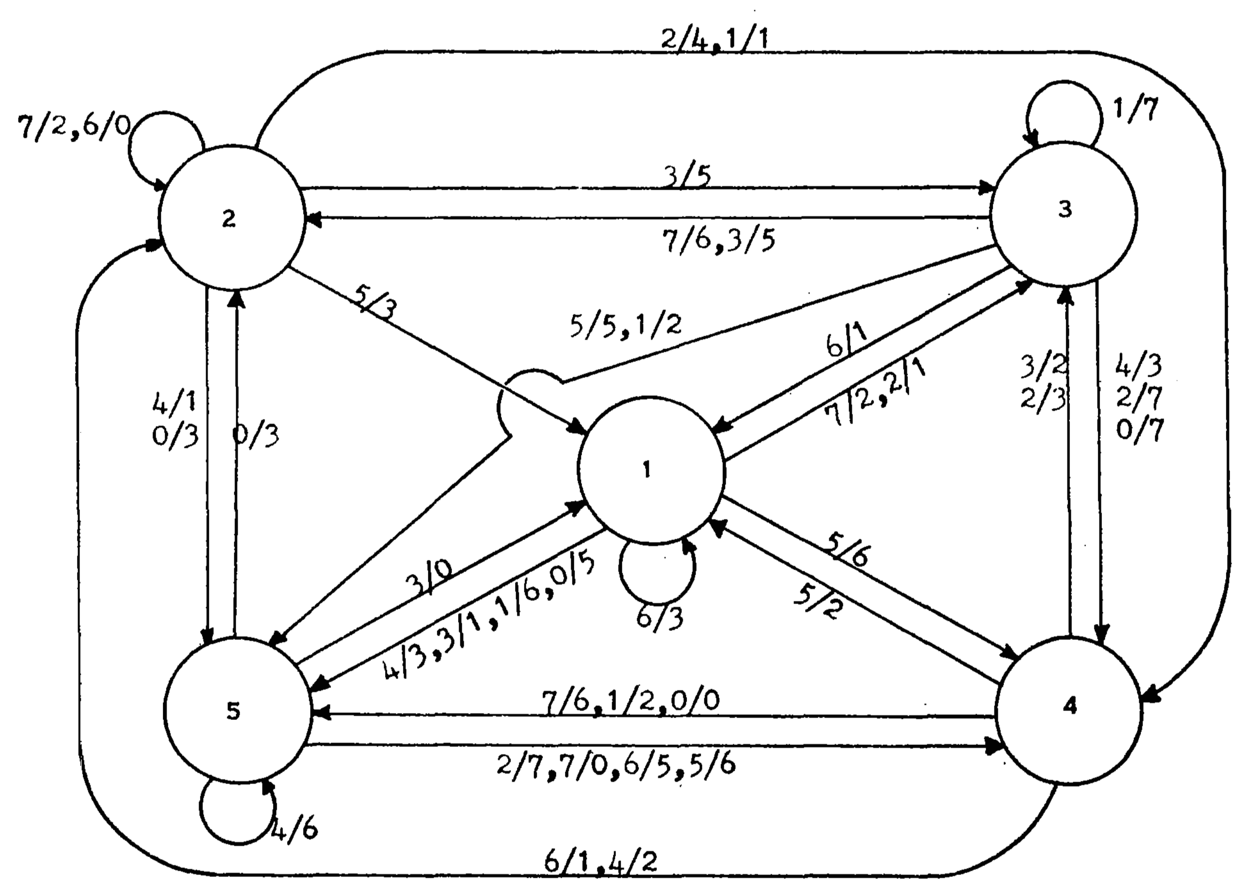
It is of interest to examine each of the predictor-machines as representations of the periodic properties of the environment. The <u>characteristic cycle</u> for any finite-state machine is found by starting it in its initial state together with the first symbol of the recall then driving it by each of its successive output symbols until the output sequence is periodic. All of the characteristic cycles in Experiment 2 were eight symbols in length. The first 73 corresponded perfectly with the pattern of the signal but this "insight" was lost in the later predictions which were in error by one or two symbols. After the 14^{th} prediction the characteristic cycle remained 13576430.

The higher noise level of the third experiment resulted in characteristic cycles of varying length until the 26^{th} prediction. From then on until the 70^{th} prediction the characteristic cycle remained maximum number of permitted mutations has been attained. If so, a transfer is effected to σ, otherwise, a transfer is effected to β.

Consider again the test for the comparison of the number of generations since the last prediction with the maximum number of generations permitted per prediction. Assuming the maximum has not been attained a transfer is effected to a test of the number of mutations per prediction attained and the logic proceeds as before.

Unless otherwise indicated all of the following experiments started with the same arbitrary five-state machine, see Figure 10. The recall was permitted to grow with experience starting with 40 symbols before the first prediction. The penalty for complexity was chosen to be 0.01 times the number of states in that machine. Single, double, or triple mutation of each parent-machine occurred with equal probability and a maximum of 40 offspring or ten generations were permitted before each successive actual prediction.

The first set of 8-symbol experiments concerned the prediction of an environment composed of a cyclic signal created by repetition of the simple pattern 13576420 which was disturbed by increasing levels of noise. With the environment consisting only of the undisturbed signal (Experiment 1) the evolutionary technique discovered a perfect one-state predictor-machine within the first eighteen evaluated offspring. The environment for Experiment 2 was generated by corrupting this signal by the equally-likely addtion of +1 or -1 to certain symbols, these being identified by skipping a number of symbols from the last disturbed symbol in accordance with the next digit drawn from a uniformly distributed random number table. Quite arbitrarily, addition to the symbol 7 and subtraction from the symbol 0 were assumed to leave these symbols undisturbed. Thus, 82.5% of the symbols were left undisturbed.



The Arbitrary Initial 8-Symbol Machine

Figure 10

As shown in Figure 11, 59.3% of the first 81 predictions were correct, there being only 6 errors in the last 30 predictions. During the evolution 3,241 different offspring were evaluated, the predictor-machines growing in size to eight states.

The environment of Experiment 3 was obtained by disturbing the environment used in Experiment 2 once again in the same manner. Thus, 28.9% of the symbols were disturbed by ± 1, 1.5% were disturbed by ± 2; leaving 69.6% undisturbed. As shown in Figure 11, 39.5% of the first 81 predictions were correct, there being a general increase in score in the last 20 predictions. During the evolution 3,236 different offspring were evaluated, the predictor-machines growing in size to fifteen states.

The environment for Experiment 4 was obtained by disturbing the environment used in Experiment 3 again in the same manner. Thus, 37.0% of the symbols were disturbed by ± 1, 2.5% were disturbed by ± 2, and 0.1% were disturbed by ± 3; leaving 60.4% of the symbols undisturbed. As shown on Figure 11, 23.5% of the first 81 predictions were correct. During the evolution 3,214 different offspring were evaluated, the predictor-machines growing in size rather steadily to nineteen states.

The environment of Experiment 5 was obtained by disturbing a randomly chosen 50% of the symbols in the signal. Thus, 43.8% of the symbols were disturbed by ± 1 (the difference being due to the adopted rule concerning addition to 7 and subtraction from 0); leaving 56.2% undisturbed. As shown in Figure 11, 22.2% of the first 81 predictions were correct. During the evolution 3,195 different offspring were evaluated, the predictor-machines growing in size somewhat irregularly to seventeen states. It

EVOLUTIONARY PREDICTION OF THE ENVIRONMENT

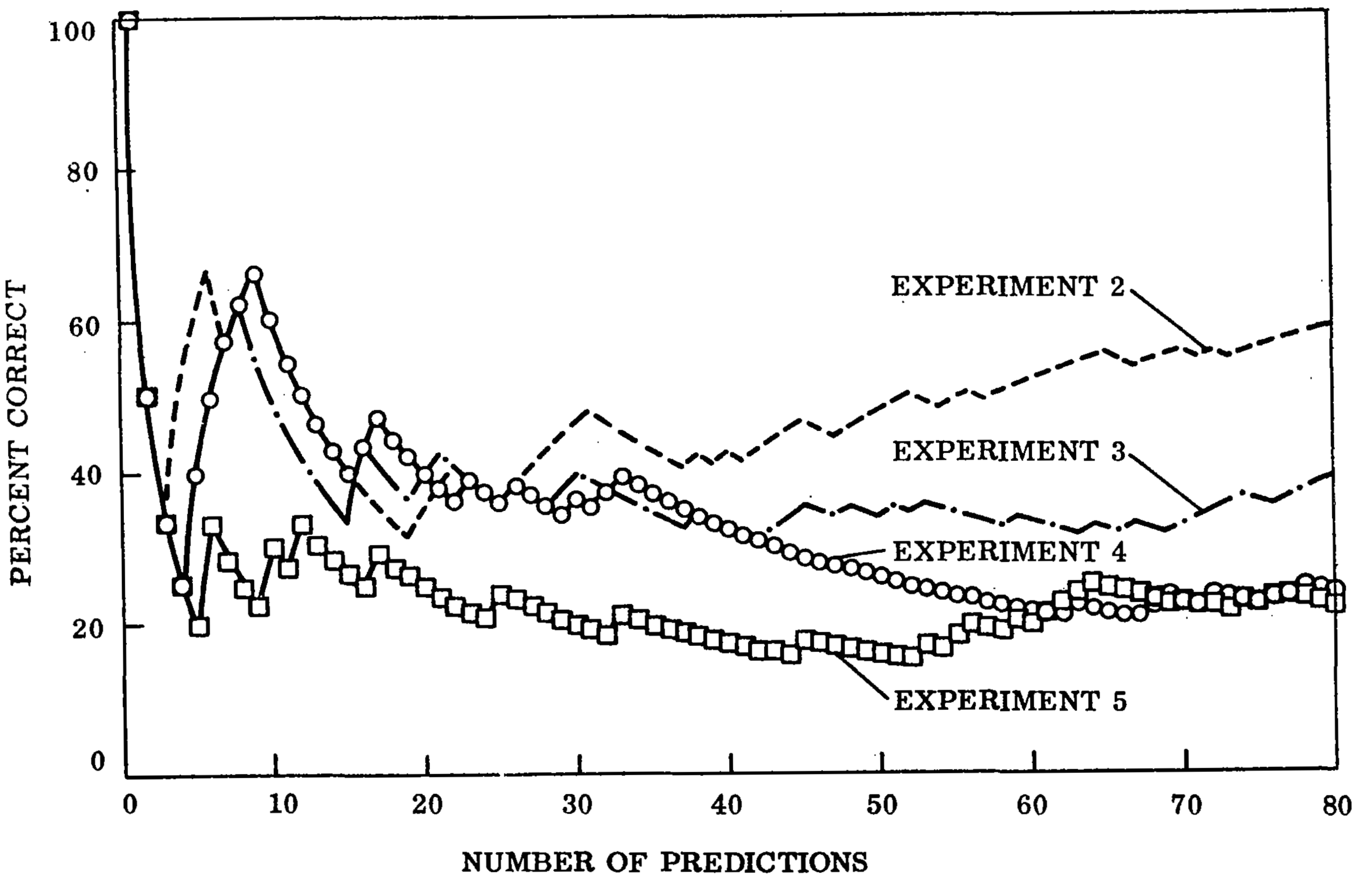


FIGURE 11

generated by the additional transduction of the new, longer recall. Slide requires not only the above additions but also the discard of the first symbol of the recall and the subtraction of the error term associated with the first transduction of each remembered machine.

After either growth or slide have been accomplished a test determines whether or not the last prediction was correct. If the prediction was correct, a transfer is effected to σ, that portion of the program predicting the next symbol to emerge from the environment utilizing the evolved machine having the lowest score. That is, a correct predictor is retained as a predictor until a prediction error occurs.

If a prediction error occurred, transfer is effected to β , that portion of the program computing the range interval for selection of a machine to be mutated.

Attention is directed to the test at location η . Assuming that the trial machine score is greater than the largest score of the stored machines, retention of the trial machine is not required and it is discarded. A test is then made to determine if the maximum permitted number of mutations per prediction has been attained. If not, a transfer is effected to α, that portion of the program wherein a machine is selected for mutation. On the other hand, if the maximum permitted mutations per prediction has been attained transfer is effected to σ.

Note again the comparison of the trial error score with that of the smallest score of the remembered machines. Assuming the trial error score is the larger, a next test determines whether or not a

EVOLUTIONARY PREDICTION OF THE SIGNAL

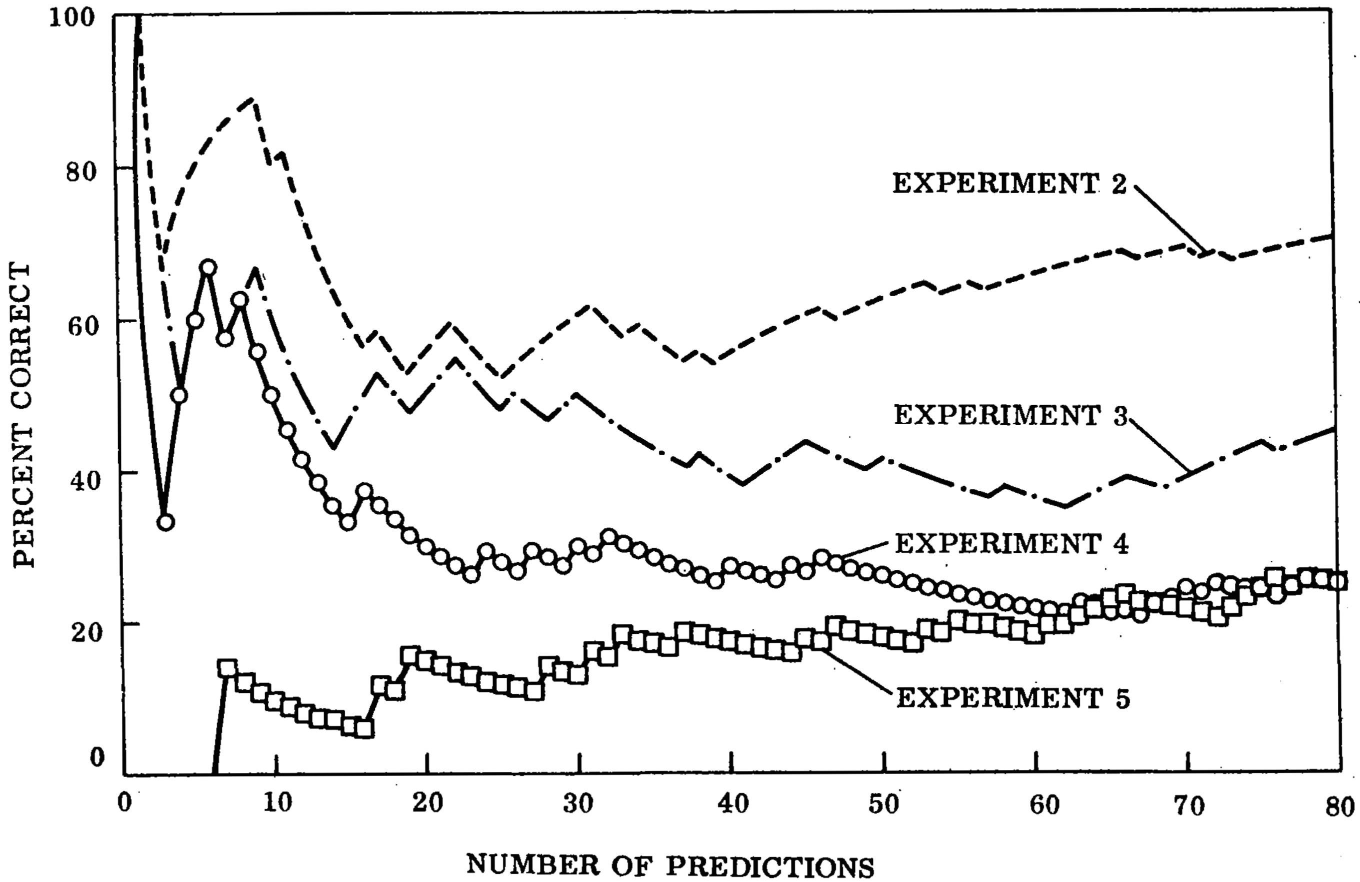


FIGURE 12

1357643113576430, this being in error one symbol out of every two cycles of the signal. As expected, the result of Experiment 4 was more erratic with the length of characteristic cycle jumping from eight to sixteen symbols and remaining the same after the 73rd prediction. Each of the last 24 characteristic cycles were 62.5% correct. Experiment 5 revealed even greater variability in the characteristic cycles. A majority of these were eight or sixteen symbols in length and reflected the basic pattern of the signal although there was little one-to-one correspondence.

The environment for Experiment 6 was generated by disturbing every symbol of the signal by +1 or -1 with equal probability. At first glance it is surprising to find the prediction of the signal improved as shown in Figure 13, but note that with the disturbance of every symbol one aspect of the randomness of the environment was removed. In essence the signal had taken on a new form...the boundaries of the original signal 24677531 or 02465310, each having equal probability at each point in time. In the first 81 predictions this new signal was properly identified 70.4% of the time. In fact, the characteristic cycle of the last predictor-machine was 02267731102665331. This can be seen to lie on the boundaries of the original signal except for one symbol of every seventeen. In order to provide a basis for comparison the percent correct prediction of the environment is also shown as well as the result of Experiment 5 with respect to the new signal.

It is natural to inquire as to the extent the prediction capability will be degraded by "wild noise" (each disturbed symbol being replaced by a randomly chosen symbol from the input alphabet). The environments of Experiments 7 and 8 were generated by imposing this kind of disturbance on the original signal once and twice, respectively. As expected,

COMPARISON OF EXPERIMENTS 5 AND 6

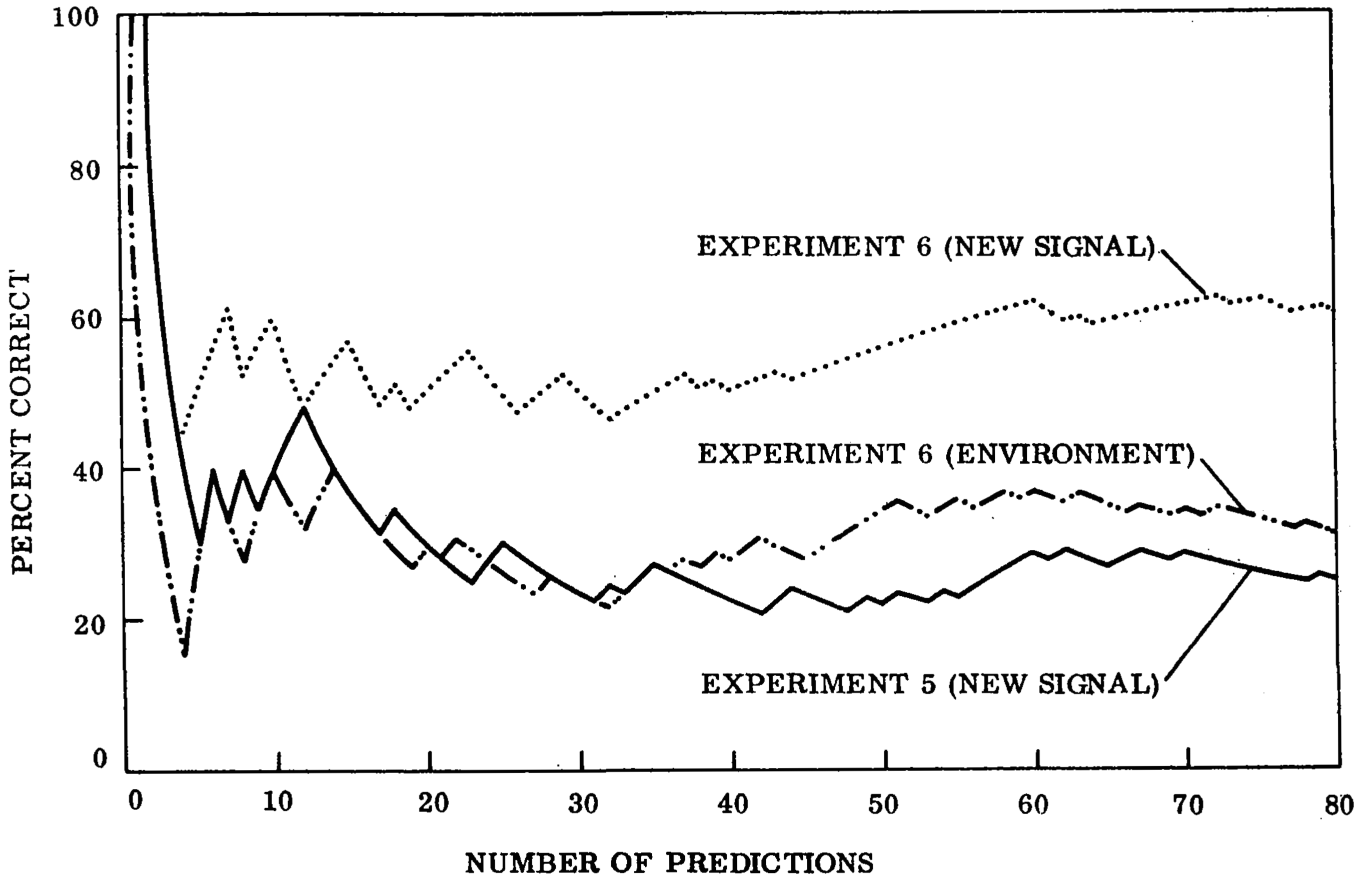


FIGURE 13

the ability of the evolutionary program to predict the environment, as shown in Figure 14, was somewhat poorer than in the comparable Experiments 2 and 3. Figure 15 indicates the degree of correspondence between the sequence of predictions and the signal in these same experiments. Here again the additional degree of randomness within the noise degrades the performance. Carrying this noise to the extreme results in a perfectly random environment which cannot be predicted. Experiment 9 revealed no significant ability of the evolutionary program in its attempt to predict such an environment.

The introduction of randomness always introduces questions of repeatability. In order to examine this point Experiment 2 was repeated nine additional times, the results being shown in Figure 16. As expected, the variability is an inverse function of the score.

The second set of experiments were concerned with the prediction of purely stochastic environments. Experiment 10 required the prediction of a zeroth-order 8-symbol Markov environment generated by combining two displayed normally-distributed variables over 5-symbols each to yield the bimodal environment shown in Table 6. A sample of 269 symbols was generated, this sample having the relative frequencies shown in this same table. With a penalty-for-complexity of 0.01 per state the predictor-machines grew in size to thirty-nine states. The relative frequency of each symbol in the sequence of predictions suitably reflects the relative frequencies shown in the sample. Experiment 11 was conducted with an increase of the penalty-for-complexity in order to decrease the sensitivity of the sequence of predictions to the difference in the relative frequency of the modes. With a penalty-for-complexity

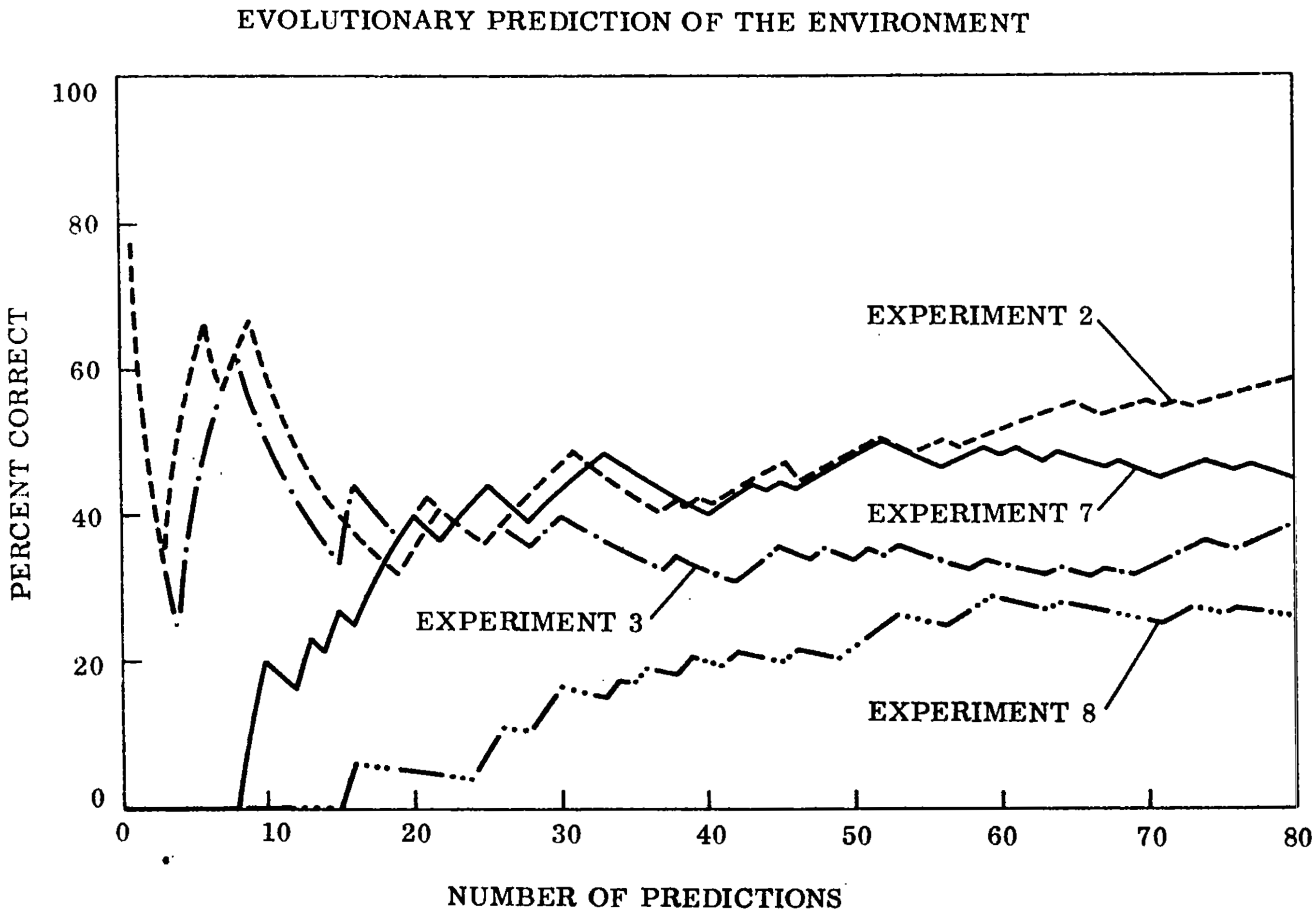

EVOLUTIONARY PREDICTION OF THE ENVIRONMENT
PERCENT CORRECT
NUMBER OF PREDICTIONS
EXPERIMENT 2
EXPERIMENT 7
EXPERIMENT 3
EXPERIMENT 8
0
20
40
60
80
100
0
10
20
30
40
50
60
70
80

FIGURE 14

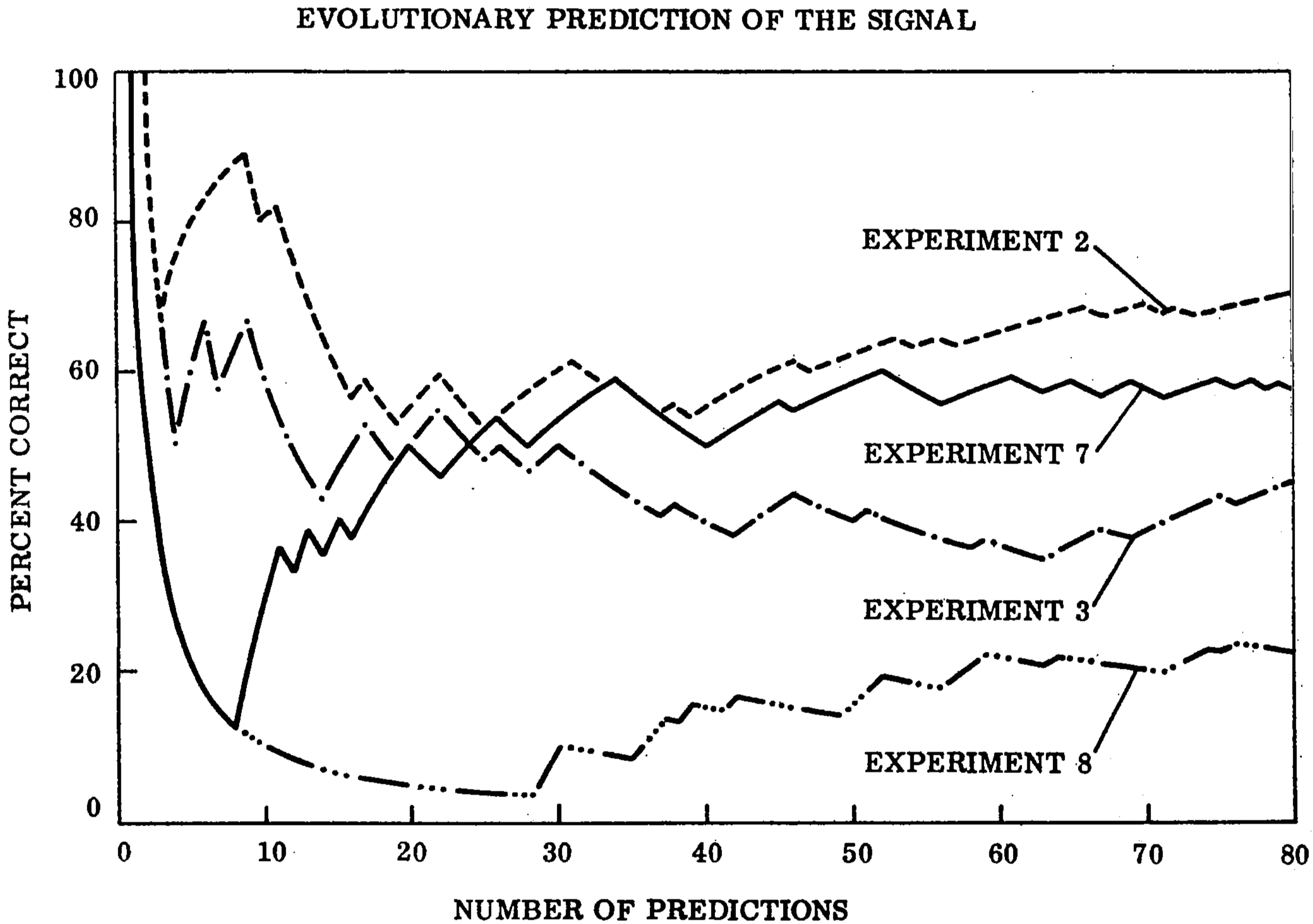

EVOLUTIONARY PREDICTION OF THE SIGNAL
PERCENT CORRECT
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
NUMBER OF PREDICTIONS
EXPERIMENT 2
EXPERIMENT 7
EXPERIMENT 3
EXPERIMENT 8

Figure 15

STATISTICS OF EXPERIMENT 2

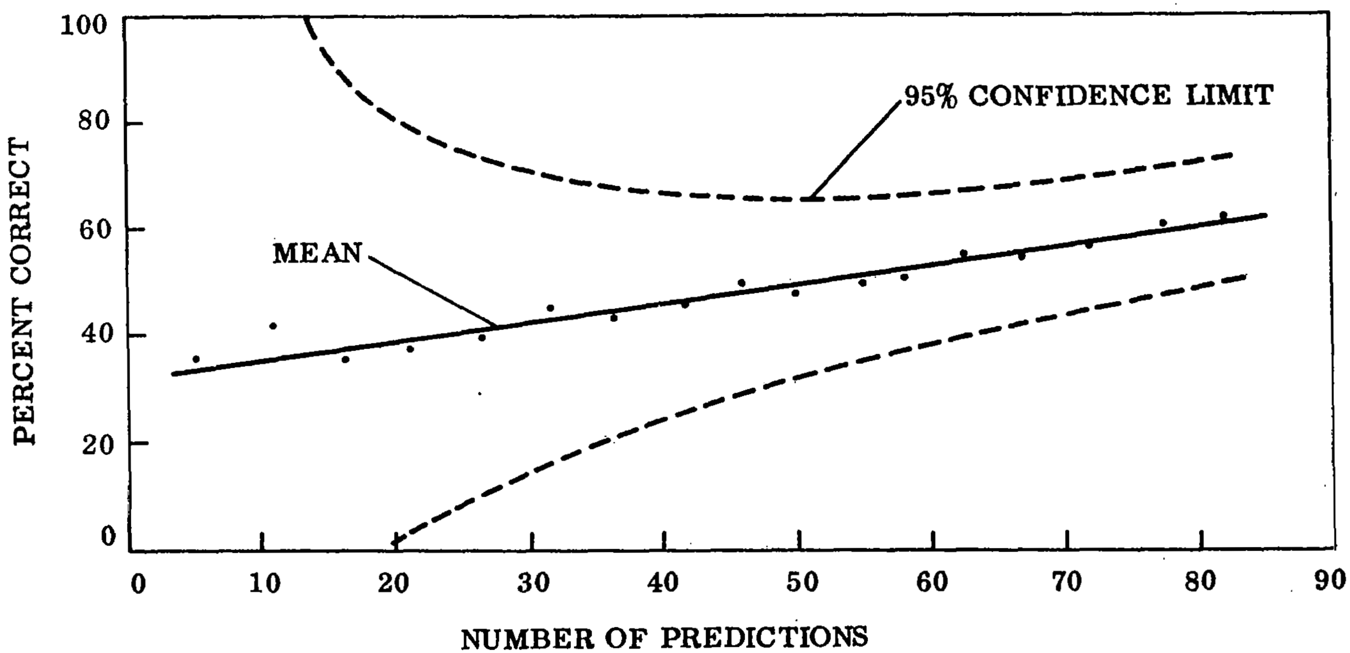


FIGURE 16

of 0.1 per state the predictor-machines grew in size only to eleven states and, as shown in Table 6, the predictions were in closer correspondence with the statistics of the environment.

Table 6

Symbol	Intended Probability	Relative Freq. of Sample	Exp. 10 Rel. Freq. of Predictions	Exp. 11 Rel. Freq. of Predictions
0	0.0116	0.00374	0.08	0.005
1	0.06785	0.0825	0.115	0.061
2	0.3413	0.299	0.223	0.303
3	0.07925	0.1050	0.058	0.081
4	0.07925	0.0787	0.017	0.015
5	0.3413	0.348	0.521	0.520
6	0.06785	0.0637	0.033	0.015
7	0.0116	0.0187	0.025	0

Experiment 12 required the prediction of a zeroth-order, 4-symbol Markov environment, the arbitrarily chosen probabilities being 0.1, 0.2, 0.3, and 0.4. This information, as prior knowledge, would dictate the continual prediction of the most probably symbol giving the asymptotic score of 40%. At the other extreme, perfectly random prediction would have an expected score of 25%. As shown in Figure 17, the evolutionary score settled between these extremes thus demonstrating the purposeful extraction of information from the previous symbols.

The first order environment of Experiment 13 was generated in such a way as to produce theoretically the results shown in Table 7. Specifically, after an initial arbitrary symbol, each next symbol of the environment was generated utilizing the relative frequency distributions

EVOLUTIONARY PREDICTION OF ZEROTH-ORDER MARKOV ENVIRONMENT

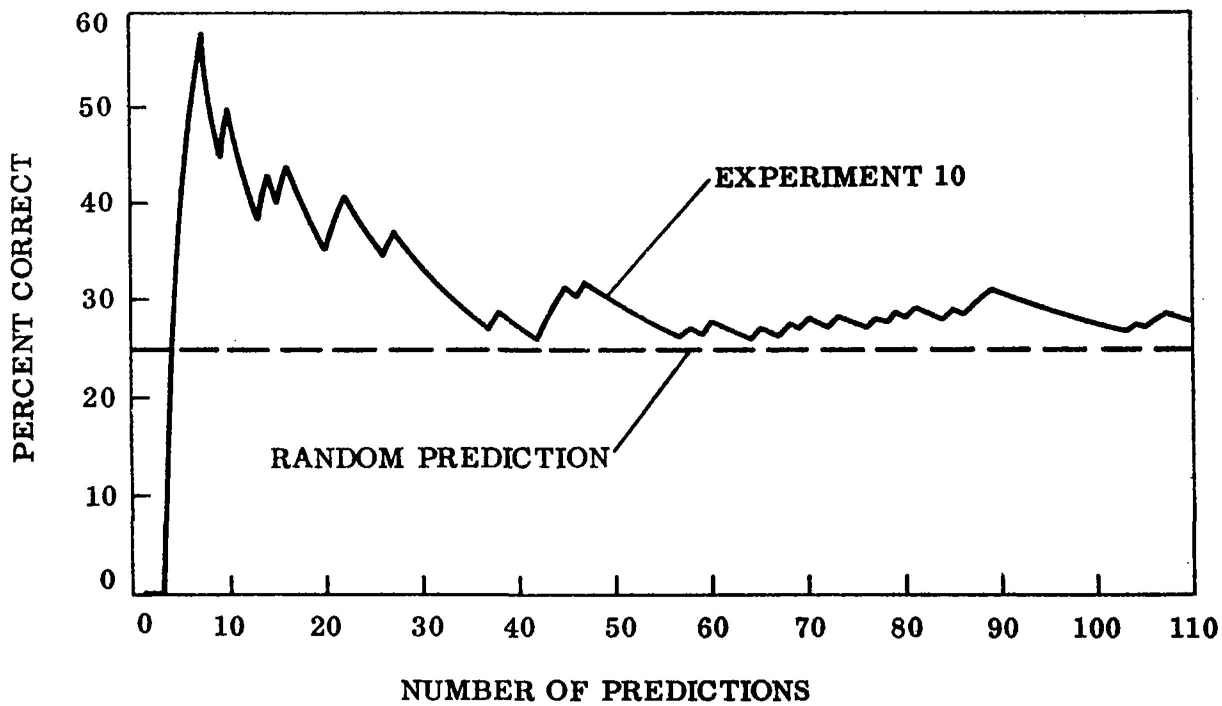


FIGURE 17

of the next symbol given a preceding symbol as indicated by the rows of the matrix shown in Table 7 where the symbols in the first row are the next symbols following the preceding symbols in the left column. The actual environment generated had the transition matrix of relative frequencies shown in Table 8.

Table 7

	0	1	2	3
0	0	0.8	0.1	0.1
1	0.1	0	0	0.9
2	0.9	0.1	0	0
3	0	0.1	0.8	0.1

Table 8

	0	1	2	3
0	0	0.822	0.071	0.107
1	0.035	0	0	0.965
2	0.915	0.085	0	0
3	0	0.077	0.862	0.061

The marginal frequencies of this environment were 0.236, 0.241, 0.249 and 0.274, respectively.

With prior knowledge that the process is first-order it would be possible to attain the score of 89.5% on the 200th prediction in the manner shown in Figure 18. <u>But even without this knowledge the evolutionary prediction technique attained this same score</u>. Analysis of the sequence of predictions revealed that at the end of this experiment the environment was properly characterized by the maximum transition probabilities of each row. Other experiments were conducted on first and second order processes with satisfactory results.

To generate a more difficult environment, the powers of 2 and 3 were rank ordered and reduced modulo 8, thus producing the sequence 12340103001 03001003010030100300... This environment was predicted in Experiment 14.

EVOLUTIONARY PREDICTION OF A FIRST-ORDER MARKOV ENVIRONMENT

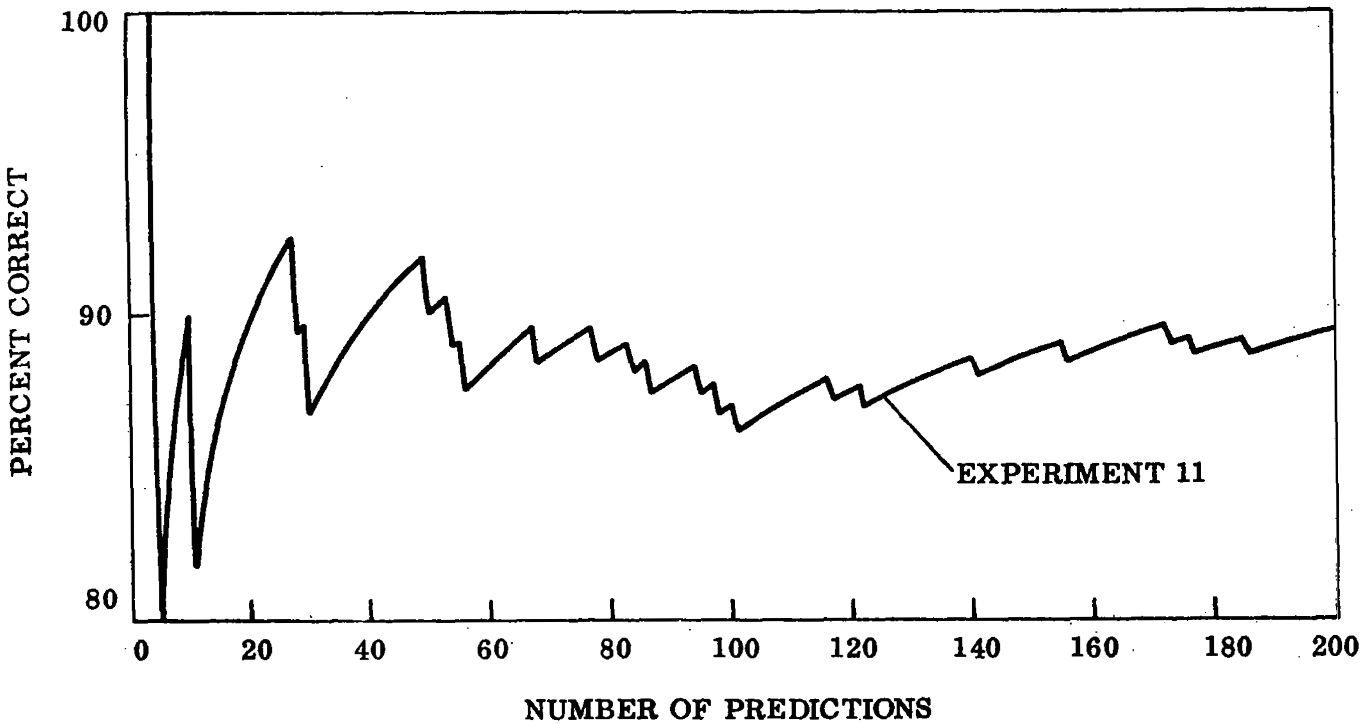


FIGURE 18

Specifically, the problem of rank ordering the powers of 2 and of 3 may be simplified by writing all of the powers in terms of a common base. For example, the powers of 3 may be written in terms of powers of 2 taking $2^{1.584925}$ as an approximation. Since $2^x > 2^v$ if, and only if, $x > v$ the problem is reduced to one of ordering the positive integers and the integral multiples of 1.584925. Since this basic exponent is in error by less than 10^{-7}, 10^7 powers of 3 may be considered before an incorrect ordering could occur.

Taking a modulus of this increasing series constrains the numbers to the alphabet available for the evolutionary program. With respect to modulo 8 the power of 2 yield the residues 2, 4, followed by 0's, while the powers of 3 are alternately 3 and 1. To see this note that $2^1 = 2 \pmod 8$, $2^2 = 4 \pmod 8$ and $2^3 = 0 \pmod 8$. For $k > 3$, $2^k = 2^3 \cdot 2^{k-3} = 0 \cdot 2^{k-3} \pmod 8 = 0 \pmod 8$. Similarly, $3^1 = 3 \pmod 8$, $3^2 = 1 \pmod 8$ so that for n even, that is $n = 2k$, $3^{2k} = (3^2)^k = 1^k \pmod 8 = 1 \pmod 8$; while for n odd, that is $n = 2k + 1$, $3^{2k+1} = 3^{2k} \cdot 3 = 1 \cdot 3 \pmod 8 = 3 \pmod 8$.

With respect to modulo 7, the powers of 2 form the repeating sequence 2, 4, 1, 2, 4, 1, ... while powers of 3 form the repeating sequence 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, ... This can be seen by noting that $2^3 = 1 \pmod 7$, $2^{3k} = (2^3)^k = 1^k \pmod 7$ and $1^k = 1 \pmod 7$; $2^{3k+1} = (2^{3k}) \cdot 2 = 1 \cdot 2 \pmod 7 = 2 \pmod 7$ while $2^{3k+2} = (2^{3k})\, 2^2 = 1 \cdot 4 \pmod 7$. Similarly, since $3^6 = 1 \pmod 7$, $3^6)^k = 1^k \pmod 7$; 3^{6k+1} $3 = 3 \pmod 7$; $3^{6k+2} = 3^{6k} \cdot 3^2 = 3^2 \pmod 7$ and $9 = 2 \pmod 7$; $3^{6k+3} = 3^{6k} \cdot 3^3 = 27 \pmod 7$ and $27 = 6 \pmod 7$; $3^{6k+4} = 3^{6k} \cdot 3^4 = 81 \pmod 7$ and $81 = 4 \pmod 7$; $3^{6k+5} = 3^{6k} \cdot 3^5 = 243 \pmod 7$ and $243 = 5 \pmod 7$.

Therefore, to form the sequence of residues modulo 8 it is only necessary to order the integers and the integral multiples of 1.584925, that is, the sequence 2, 4, followed by all 0's is inserted in successive positions corresponding to the integers while the repetitive sequence 3, 1, . . . is inserted in the successive positions corresponding to the multiples of 1.584925. Similarly, for modulo 7 the integer sequence 2, 4, 1, 2, 4, 1, . . . is interlaced with the sequence 3, 2, 6, 4, 5, 1, 3, . . . which corresponds to the multiples of 1.584925. The resulting sequences are not periodic although sub-periods are embedded throughout. This lack of periodicity results since in higher multiples of 1.584925 the successive decimal digits influence the integral part and thus upset any pattern imposed by the more significant digits.

After the first 300 predictions (Experiment 14) the percent correct score reached 88.7. 1,401 different offspring were evaluated using an "all-or-none" error matrix. (All off-diagonal elements have a value of 7 and all main diagonal elements a value of 0.) The predictor-machines were generally of six states. In order to avoid the reduction to 3 symbols in the latter portion of the sequence, the powers of 2 and 3 were rank ordered reduced modulo 7 yielding the sequence 12341226414254112431224614245121431 22461 . . . After the first 216 predictions (Experiment 15), the percent correct score was 56.6, this being found through the evaluation of 3,671 offspring, which were generally of about twenty-one states. The score for the last fifty predictions was 78%. Certainly the prediction capability was far better than chance would yield. Prediction of this sequence based on the most probable symbol up to each point in time yields a score of only 8.5%

With the capability of the evolutionary program demonstrated, pilot experiments were conducted in order to predict solar flare data as found in IGY Solar Activity Report Series, Number 17, dated May 1, 1962, (World Data Center A, High Altitude Observatory, University of Colorado, Boulder, Colorado). Analysis of these data reveals that there were about 1.75 importance 3 flares, 7.75 importance 2+ flares and 63.3 importance 1+ flares for every importance 3+ flare occurring in 1957. Relative to this same datum, there were 0.5 importance 3+ flares, 1.125 importance 3 flares, 5.375 importance 2+ flares, 7 importance 2 flares, 26.75 importance 2^- flares and 42 importance 1+ flares in the year 1958. A suitable data base was arbitrarily taken to include flares of importance 2^- and greater, these being encoded into an 8-symbol language by doubling the importance number and counting + as an additional unit. The environment contained no temporal information other than the order of flare occurrence.

In the first experiment, a magnitude-of-the-difference error-weight payoff matrix was used. That is to say, a prediction of the importance of each next flare is based upon the importance of the preceding flares and their temporal order with the penalty for incorrect predictions being equal to the magnitude of error. Forty flares were taken as the initial recall. Of the first 300 predictions 57.7% were correct, these predictions being made by a one-state machine which quickly evolved to demonstrate the statistical dominance of importance 2^- flares and that importance 3 flares are best predicted by persistence (that is, they may be expected to be followed by another flare of similar importance).

In order to evaluate the significance of this result, another computer program was written which would predict these same data on the basis of the maximum marginal probability, the maximum first-order conditional probability, the maximum second-order conditional probability, and any of these with the program choosing that one which had highest score up to that time. After the first 300 predictions, these conventional techniques showed scores ranging from 55.4% up to 56.1%; this highest score being achieved by the maximum marginal predictor. Thus, it would seem that the evolutionary program had discovered a suitable logic for prediction in that its score was slightly superior than this score. Unfortunately, this ability is not considered to be significant because no particular capability has been demonstrated for the successful prediction of the more important flares.

To improve this situation, the error-cost matrix was made to reflect a linear weighting favoring the more important flares, as shown in Table 9. Another experiment was then conducted.

Table 9

	2^-	2	2+	3	3+
2^-	4	5	6	7	7
2	5	3	4	5	6
2+	6	4	2	3	4
3	7	5	3	1	2
3+	7	6	4	2	0

As a result, within the last 50 predictions, 18 of the 27 importance 2^- flares, 2 of the 12 importance 2 flares, one of the 8 importance

2+ flares were properly predicted. Neither of the 2 importance 2 flares were properly predicted and the only importance 3+ flare was properly predicted. It would appear that the evolutionary program was still not placing sufficient weight on the relative importance of correctly predicting the large flares.

To further improve the situation, another experiment was performed on these same data but with the error cost matrix shown in Table 10.

Table 10

	2^-	2	2+	3	3+
2^-	9	10	11	12	13
2	11	4	5	6	7
2+	12	6	2	3	4
3	13	7	4	1	2
3+	14	8	6	3	0

A penalty of 0.01 was used. Analysis of the results reveal that 26.8% of the 2^- flares were correctly predicted, 45.1% were incorrectly predicted as being of importance 2, 24.6% were incorrectly predicted as importance 2+, 1.7% were incorrectly predicted as being of importance 3 and 2.3% were incorrectly predicted as importance 3+. Of the importance 2 flares which occurred, 23.1% were incorrectly predicted as importance 2^-, 57.0% were correctly predicted, 18.5% were incorrectly predicted as being 2+, and 15.4% were incorrectly predicted as being 3. Of the importance 2+ flares which occurred, 20.4% were incorrectly predicted as being of importance 2^-, 50% were incorrectly predicted as being of importance 2, 27.3% were correctly predicted, and 2.3% were incorrectly predicted as being of importance 3. Of the importance 3 flares which occurred, 50% were incorrectly predicted as being 2^-, 25% were incorrectly predicted as being 2 and 25% were correctly predicted. Of the importance 3+ flares which occurred, 16.7% were incorrectly predicted as being 2^-, 50% were incorrectly predicted as being 2 and 33.3% were incorrectly predicted as being 2+.

It is apparent that further adjustment of the error cost matrix is desired; however, the adjustment of the error-cost matrix already achieved demonstrates a significant improvement in that the predominance of the least importance flares is mitigated as a factor in determining the sequence of predictions. In general, this revised description of the goal generally encourages false alarms as opposed to missed flares. No further effort was made to improve the prediction of these data in view of the practical importance of predicting both the magnitude and the time of occurrence of each next flare.

At this point attention was therefore focused upon the problem of finding a more suitable data base, one which might increase the probability of correct prediction of large flares by considering each previous flare only within the context of directly relevant data . An attempt was made to group the above referenced flare data by location into the associated plage but a suitable quantitative criterion was unavailable. Upon hearing of this difficulty Miss D. Trotter of the High Altitude Observatory, Boulder, Colorado, furnished a series of report entitled "Solar Activity Summary" (HAO-42 through HAO-54, except for HAO-48), covering the time period from January 13, 1959 to January 7, 1961. These reports describe active regions and present tabular data concerning major flares and the accompanying radio noise. Unfortunately, the regions are significantly large so that they may simultaneously contain a number of separate plages. Miss Trotter was kind enough to separately identify the McMath numbers for the plages within each region; however, it would be difficult if not impossible to separate the flare data contained in these reports into the associated plages in successive rotations. Further, few flares are reported within each specific listing.

As a result of a request to Mr. D. Robbins of NASA/Houston, the draft of a forthcoming report on the solar activity for Calendar Year 1958 was received and copied (the original being returned to him). This report catalogues plage data, flare data, sun spot data, and additional terrestrial effects. Analysis of these data allowed the compilation of the data contained in Table 11 which lists the plage family, individual plage data, and flare data within that plage. Specifically, the plage family number designates a particular plage which may survive several rotations of the sun. For example, plage family 2 was first identified with McMath plage #4355 which was later identified as plage 4440, then as plage 4445, and lastly as plage 4483. Throughout this history from January 11.5 through April 4.5 (CMP Gr. Day) a total of 12 flares were separately identified, these ranging in importance from 1 through 3. Most of the families were less suitably identified in terms of the number of flares even though some existed for many more rotations.

In order to attack this problem, attention was turned to "National Bureau of Standards List of IGY flares with Normalized Valued in Importance And Area" by C. S. Warwick, Series #17, dated May 1, 1962. This report covers the same time period as the above reference data and was examined in detail in the hope that a more complete listing of the associated flares could be obtained. Unfortunately, decisions on many of the listed flares were difficult to make in that no obvious basis was avail-

Table 11

Plage Family	McM. Plage No.	CMP Gr. Day	Mean Long.	Mean Lat.	Ave. Int.	Max. Area	No. Flares	Age In Rotation	Identi- fication	Gr. Day	Beg. UT	End UT	Max. UT	Imp.	Position
	PLAGE DATA									FLARE DATA					
1	4347	7.0	78°	N13	3	13000	22	3	4296	Jan 07					
2	4355	Jan 11.5	19°	S12	3.5	9000	33	1	NEW	Jan 07	1820	1939	1832 1842	2	S18E39
	4355									Jan 15	1640	1757	1642	2+	S13W58
	4400	Feb 8.5	24°	S12	3.5	25000	91	2,3	4355 4356 4360 4362	Feb 09	1330	1501	1341	1+	S20W02
	4400									09	2108	2302	2142	2+	S12W14
	4400									10					
	4400									10	1320	1411	1332	2+	S13W64
	4400									10	1900	2030	1911	1+	S14W64
	4445	Mar 7.5	15°	S15	3	10000	45	3,4	4400	Mar 03	1005	1411	1020	3	S16E60
	4445									Mar 05	0500	0632	0540	3	S12E46
	4445									Mar 14	1454	1541	1507	2	S21W85
	4483	Apr 4.5	6°	S22	3	5000	33	4,5	4445	Apr 02	1951	2025	1954	1+	S15E23
	4483									Apr 06	1929	2025	1940	1	S15W27
	4483									06	0301	0408	0309	1	S17W44
3	4368	16.5	313°	S07	3	6000	3	1	NEW	Jan 17 18	2253	2335	2256	1	S11W32
4	4370	17.5	300°	N15	2.5	6000	26	1	NEW	Jan 16	1414	1447		1+	N18E23
5	4377	20.5	261°	S14	2	3000	6	3	Part of 4319	Jan 16	2255	2347	2306	2	S15E47
	4372	20.5	261°	S24	2	3500	10	4,5	4318 4319	Jan 25	0915	1107	1005	3	S23W70
6	4426	22	193°	S16	3	6000	6	7	4382	Feb 26	0527	0632	0550	2	S18W61
7	4436	25	153°	S12	3.5	1400	15	1	NEW	Mar 01	0905	1007	0917	3	S11W45
8	4444	7	22°	N31	2.5	4000	18	1	NEW	Mar 09	1540	1740	1546	2	N34W32
	4484	Apr 4.5	6°	N30	3	9000	48	2	4444	Mar 29	1339	1410	1343	2	N35E78
	4484									Mar 30	0045	0123	0108	2	N35E74
	4529	May 2	3°	N22	3	14000	60	3,5	4484 4485	May 2					
	4529									May 5	2025 2032	2115 2115	2035 2037	1+ 1+	N24W50 S15W39
	4578	30	352°	N24	3	8000	47	4,6	4529	Jan 04	2147	2356	2152	2	N14W58
	4578	May 30								Jan 05	0835	0956	0850	2+	N15W65
	4578									06	0436	0614	0448 0505	2	N16W78

Table 11 (Cont'd.)

9	4449	12	316°	N12	2.5	9500	31	3	4410	Mar 11	0030	0042	0034	1	N11E12
	4449									Mar 12	0024	0233	0037	2+	N08E02
10	4465	22.5	177°	N21	3	7000	66	1	NEW	26	0036	0040		1	N21W50
	4465									Mar 28	1833	1922	1838	2	N20W85
	4465'									29	1630	1637	1632	1+	N21W90
11	4469	Mar 25	144°	N25	3	6000	19	1	NEW	29	1447	1507	1449	1	N26W70
12	4476	28.5	98°	S12	3.5	15000	90	2	4442a	Mar 23	0947	1445	1005	3+	S14E78
	4476									Mar 30	0944	1421	1000	2+	S16W20
	4476									Mar 31	0005	0036	0014	2	S17W22
	4476									Mar 31	0038	0130	0052	2	S08W23
	4476									Mar 24	1607	1643		1	S15E57
	4476									Mar 25	0557	0626	0603	2	S15E50
	4476									Mar 27	1534	1710	1552	2+	S16E23
	4476									Mar 28	1703	1904	1714	2+	S15E09
13	4478	30	78°	S22	3	6000	29	2	4438	Mar 28	1030	1152	1038	2	S24E26
	4478									Mar 29	2042	2131	2047	2	S24E21
	4478									Mar 29	1819	1915	1823	2	S24E08
14	4493	9.5	300°	N16	3	5000	34	2	4453	Apr 07	1010	1215	1025	3	N14E32
15	4508	Apr 21.5	141°	S21	3.5	7500	30	1	NEW	Apr 21					
16	4519	26	349°	N09	3.5	6000	6	1	NEW	Apr 30	b 1932	2015	1940	1+	N10W50
17	4530	3	82°	S15	3.5	11000	77	1	NEW	Apr 30	a 1930	2005	1940	1+	S17E27
	4530									May 1	2115	2241	2130	3	S18E15
	4530									May 5	0356	0457	0415	3	S18W29
18	4548	15.5	184°	S21	3.5	13000	49	2	4516	May 15					
	4598	June 11.5	187°	S20	3	7000	10	3	4548	June 05	1615	1656	1631	2+	S18E69
	4636	06	196°	S22	3.5	8000	42	4	4598	June 04	1712.5	1722.5	1717.5	1-	S23E32
											1747.5	1755	1750	1-	S23E32
19	4596	10	207°	N28	3.5	10000	30	1	NEW	June 10					
	4634	07.5	203°	N28	3	9000	23	2	4596	June 07	0020	0414	0110	3+	N25W08
	4634									June 12	2317	2330	2330	1	N26W78
20	4597	10	207°	N43	3	7000	77	1	NEW	June 10					
21	4607	June 18	101°	N12	3.5	7000	52	3	4563	June 14	2112	2146	2118	1	N14E36
	4607									June 19	0940	1210	1010	3	N14W21

Table 11 (Cont'd.)

22	4618	26.5	1°	S16	3	11000	15	3	4579	June 26				
23	4622	29	315°	S19	3.5	7000	38	2	4581	June 29				
24	4659	July 26.5	311°	S19	3	20000	112	3	4622	July 29	0259	0408	0304	3	S14W44
	4659									July 29	0458	0526	0458	1	S14W38
	4659									July 30	1523	1637	1530	2	S13W64
	4659									July 31	1058	1150	1122	1+	S13W77
	4659									Aug 2	1840	1851	1841	1-	S14W90
	4710	22	321°	S15	3	9000	12	4	4659	Aug 22	1032	1210	1043	1+	S14W90
	4712	Aug 24	294°	S18	3	7500	15	4	4659	Aug 28	1025	1045	1030	2	S18W65
	4765	19.5	305°	S18	3.5	17000	58	5	4712 4710	22	0730	0910	1750	2	S19W42
25	4623	29.5	309°	N12	3	12000	16	1	NEW	June 26	0245	0517	0306	2+	N10E49
	4623									June 27	0254	0405	0308	1+	N10E37
26	4630	July 05	236°	N24	3	11000	39	1	NEW	July 03	0041	0114	0050	1+	N30E37
	4630									July 04	0513	0534	0517	1+	N29-E26
	4667	Aug 02	225°	N27	3.5	15000	27	2	4630	July 04	0409	0610	0435	3	N30W31
27	4646	18	64°	N09	3	5500	40	1	NEW	July 18					
28	4651	21	24°	N22	2.5	8500	21	1	NEW	July 19	1905	2030	1908	2+	N23E13
29	4665	31	252°	N04	3.5	12000	30	2	4631	July 25	0000	0128	0043	2+	N10E35
30	4670	04	199°	S09	3.5	5000	31	1	NEW	Aug 04	2112	2127	2114	1	S07W10
	4722	31	202°	S09	3.5	12000	54	2	4670	Aug	b 1028	1047	1030	1	S09E38
31	4686	12.5	86°	S13	3.5	11000	72	2	4653	Aug 07	1457	1700	1508	3	S16E71
	4686									Aug 14	2137	2225	2203	1+	S14W36
	4686									Aug 16	0433	0831	0440	3+	S14W50
	4739	08	96°	S20	3	14000	17	3	4684 4686	Sept 07	1639	1726	1643	2	S32E18
32	4708	22	321°	N18	3.5	8000	60	3	4657	Aug 19	2118	2411	2256	2	N18E26
	4708									Aug 20	0042	0128	0045	2+	N16E18
	4708									Aug 22	1428	1717	1450	3	N18W10
	4708									Aug 26	0005	0124	0027	3	N20W54
	4756	17.5	331°	N17	3	9000	26	4	4708	Sept 17					
33	4741	Sept 09	53°	S07	3	7000	57	1	NEW	Sept 02	2102	2141	2105	1+	S08E84
	4741									Sept 14	0622	1030	0835	2+	S10W80

Table 11 (Cont'd.)

34	4743	9.5	76°	N17	3	6000	17	1	NEW	Sept 09					
35	4750	14.5	11°	S10	3	9000	20	2	4703	Sept 18	0728	0938	0830	3	S11W53
36	4764	20	298°	N23	3.5	6000	22	3,4	4711	Sept 20					
37	4777	24	245°	N30	3	3000	7	1	NEW	Sept 28	2046	2108	2054	1-	N32W66
38	4781	30	166°	S10	3.5	7500	22	1	NEW	Oct 02	2143	2201	2148	1	S06W38
39	4806	Oct 10.5	27°	N13	3.5	3000	6	2	4748	Oct 08	1510	1528	1522	1-	N12E25
40	4826	20.5	255°	S02	3.5	6500	50	1	NEW	Oct 21	2316	0127	2330	2+	S04W22
	4826									Oct 24	1432	1801	1457	2+	S05W57
41	4829	22.5	229°	S10	3	9000	36	2	4779	Oct 22					
	4877	18	240°	S12	3	11000	4	3	4829	Nov 14	0036	0207	0046	3	S19E51
42	4838	Oct 27.5	163°	S30	2	2000	5	1	NEW	Oct 23	1655	1803	1728	1	S32E50
43	4849	Nov 3.5	71°	S15	3	6000	49	1	4817	Nov 03					
	4897	Nov 30.5	75°	S18	2.5	12000	31	3	4849	Nov 30					
	4934	28	73°	S17	3.5	10000	48	4	4897	Dec 23	0545	0730	0624	2+	S15E66
	4934									Dec 31	1656	1741	1703	3	S18W54
44	4851	3.5	71°	N08	3	5500	11	1	NEW	Nov 03					
45	4883	24.5	154°	S12	3.5	12000	34	1	NEW	Nov 24	1607	1907	1621	3	S11W08
46	4884	25.5	141°	N22	3	6000	33	1	NEW	Nov 27	1857	1909	1859	1	N18W12
	4884									27	2354	0020	2356	1	N19W19
47	4898	Dec 02	55°	N15	2.5	7500	8	3,4	4854	Nov 30	2240	2308	2250	1-	N13E22
	4898									Dec 09	1642	1735	1654	1	N10W90
48	4911	Dec 09.5	316°	N16	3	9000	31	1	NEW	Dec 09					
49	4913	12	283°	S03	3.5	9500	69	3	4873	Dec 10	0219	0306	0221	2	N01E20
	4913									Dec 10	1312	1514	1318 1428	1	S03E18
	4913									Dec 11	1545	1612	1550	1-	S02E00
	4913									Dec 11	1640	1707	1647	1-	S02E10
	4913									Dec 11	1705	1745	1720	1-	S02E00
	4913									Dec 11	1802	1842	1812	2	S02E00
	4913									Dec 11	1850	1917	1857	1-	S02W02
	4913									Dec 11	1930	2012	1939	2	S02E02
	4913									Dec 12	1229	1547	1304	2+	S03W08
	4913									Dec 15	1535	1550	1538	1-	S04W49
	4913									Dec 17	1040	1115	1041	1	S04W82
50	4905	Dec 5.5	9°	S07	2.5	3000	5	1	NEW	Dec 11	1740	1755	1745	1-	S07W88
51	4919	15	244°	N10	3	5000	10	1	NEW	Dec 17	1855	1927	1900	1+	N07W35
52	4936	29	59°	N16	3.5	15000	30	4,5	4898	Dec 29					

able for determining the particular extent and shape of the plage at the time of the considered flare. To illustrate, flare #5464 shown to occur at 58 01 14 0732 17S34W was judged to fall outside of the relevant plage domain even though it is only about 10 degrees away from the expected position.

Table 12 indicates those flares which appear to have been associated with the second plage family. All of the flares indicated to exist within this family in Table 11 were not exactly identified. Further, certain discrepancies were noted between the listings furnished by Robbins and Warwick. In any event, these data appeared to be unsuitable as a basis for prediction because of the significant absence of that half of the information which is generated on the far side of the sun. In view of the present difficulty in obtaining such information, it was considered more suitable to examine data which occurs within a single crossing of the solar disc in greater detail in the belief that information derived during passage over the eastern hemisphere will prove helpful in the prediction of proton events which might occur as a result of flare activity in the western hemisphere. The scarcity of such events in the eastern hemisphere indicates the need for additional data in terms of other parameters such as plage age, shape, intensity, the nature and number of sunspots, magnetic intensity, etc. A search was made for such data on a daily basis or even for each 6 hour time interval. A review of the literature revealed a recent Russian book on the forecasting of solar activity. As a matter of interest the table of contents, chapter 3, and the conclusion were translated and are included in this report as an addendum.

At the suggestion of Dr. C. Warwick, contact was made with Dr. F. Ward of AFCRL, who indicated that the Air Force is presently devoting about seven

Table 12

FLARE NO.	YR	MO	DA	FIRST BEG	LAST END	COR IMP	AREA SQ DEG	MEAN LAT	MEAN CMD
5296	58	01	07	0304	0313	1-	.7	18S	41E
5297		"		0315	0322	1-	1.5	16S	45E
5298		"		0413	0434	1-	1.5	16S	44E
5299		"		0858	0905	1-	-	17S	45E
5303		"		1820	1939	2-	8.6	16S	39E
5311	58	01	08	0141	0151	1-	1.5	13S	48E
5315		"		0751	0800	1-	1.0	14S	42E
5318		"		1731	1746	1-	.8	18S	44E
5322		"		1935	1941	1-	.3	12S	33E
5323		"		2008	2013	1-	.4	16S	41E
5336	58	01	09	1029	1038	1-	3.5	17S	32E
5337		"		1116	1143	1-	1.4	19S	29E
5343		"		1506	1524	1	2.8	10S	25E
5344		"		1525	1552	1-	2.4	13S	25E
5348		"		1546	1552	1-	.4	12S	23E
5352		"		1930	1947	1-	.6	15S	28E
5353		"		2142	2202	1-	.5	11S	20E
5365	58	01	10	0843	1000	1	1.8	16S	17E
5373		"		1106	1151	1-	2.0	15S	11E
5375		"		1321	1342	1-	1.3	14S	18E
5381		"		1628	1644	1-	.6	15S	07E
5387		"		2120	2145	1-	2.6	13S	11E
5388		"		2212	2222	1-	.7	15S	04E
5397	58	01	11	1657	1717	1-	1.4	12S	01W
5398		"		1722	1742	1	2.6	16S	03W
5399		"		1810	1836	1-	-	15S	01W
5401		"		1902	1947	1	4.6	11S	04W

Table 12 (continued)

FLARE NO.	YR	MO	DA	FIRST BEG	LAST END	COR IMP	AREA SQ DEG	MEAN LAT	MEAN CMD
5402	58	01	12	0630	0651	1	2.9	18S	09W
5403		"		1236	1248	1-	2.2	16S	15W
5410		"		1424	1527	1-	1.4	11S	12W
5411		"		1443	1453	1-	.4	18S	14W
5417		"		1927	1935	1-	.6	17S	16W
5439	58	01	13	2037	2047	1-	1.1	11S	40W
5444		"		2215	2232	1-	.6	11S	41W
5446		"		2232	2241	1-	.6	14S	42W
5450	58	01	14	0034	0041	1-	.4	15S	43W
5454		"		0140	0148	1-	.4	15S	43W
5457		"		0230	0238	1-	.6	16S	52W
5460	"	"		0301	0306	1-	.6	15S	44W
5462		"		0543	0608	1-	.6	14S	44W
5465		"		0955	1010	1-	-	15S	44W
5468		"		1351	1400	1-	1.5	13S	41W
5470		"		1540	1755	1+	3.6	16S	43W
5472		"		2142	2215	1	1.8	18S	42W
5473	58	01	15	0056	0106	1	.5	11S	58W
5476		"		0500	0641	1	3.4	13S	53W
5478		"		0747	0755	1-	1.3	13S	55W
5481		"		1017	1032	1	1.6	13S	54W
5485		"		1640	1757	3-	8.5	14S	58W
5489		"		2056	2102	1-	.8	12S	65W
5490		"		2106	2118	1	.6	10S	66W
.				.		.		.	
.				.		.		.	
.				.		.		.	

Table 12(continued)

FLARE NO.	YR	MO	DA	FIRST BEG	LAST END	COR IMP	AREA SQ DEG	MEAN LAT	MEAN CMD
5890	58	02	09	1330	1501	1+	6.7	20S	01W
5900	58	02	10	0834	0845	1-	-	22S	08W
5908		"		1256	-	1-	-	21S	11W
5913		"		1540	1617	1-	2.0	22S	14W
5916		"		1900	1907	1-	.4	17S	23W
5926	58	02	11	0745	0817	1-	1.9	20S	25W
5932		"		0915	0919	1-	.6	17S	38W
5934		"		0941	1035	1-	.9	19S	46W
5985	58	02	13	1018	1110	1+	3.8	18S	49W
5997	58	02	14	1223	1231	1-	.7	16S	57W
6003	58	02	15	0158	0216	1	-	15S	67W
6006		"		0711	0732	1	.8	22S	72W
5896	58	02	09	2108	2302	2	13.5	11S	15W
5916	58	02	10	1900	1907	1-	.4	17S	23W
5937	58	02	11	1319	1342	1-	1.0	23S	25W
5938	58	02	11	1342	1542	1+	5.4	22S	27W
5958	58	02	12	0937	1012	1+	4.9	21S	35W
5902	58	02	10	0917	0918	1-	-	13S	69W
5909		"		1320	1411	2-	3.5	13S	65W
5915		"		1900	2030	1+	3.1	12S	64W
5928	58	02	11	0820	0836	1	.4	13S	80W
5946		"		2237	2247	-	.8	18S	86W
.				.		.		.	
.				.		.		.	
.				.		.		.	

men full time to the task of collating, coding and keypunching astrophysical data as this might relate to solar flares. This work is in conjunction with that of National Bureau of Standards and information is being drawn from observatories here and abroad. It was anticipated that this information will be available in about six month's time. However, until then, Dr. Ward preferred to keep these data undisclosed and undisturbed. At his suggestion, and at the suggestion of Dr. Warwick, contact was made with Dr. R. Howard of the Mount Wilson-Palomar observatories.

A meeting with Dr. Howard revealed that although a considerable number of measurements are currently being taken on the magnetic field strength in the vicinity of solar activity, the resulting data is not considered to provide a sufficiently valid data base in view of gaps and inaccuracies. A review was made of the suggested data reduction technique in which information derived from IGY Solar Activity Report Series #12, dated 25 June 1960, "McMath-Hulbert Observatory Working List of IGY Flares" by Helen W. Dodson and E. Ruth Hedeman, was used in conjunction with the CRPL-F Part B solar-geophysical data in an effort to provide a consistent tabulation of plages in terms of their individual history. That is, the former listing provides an identification of each flare together with its McMath plage number. The latter provides an identification of the previous McMath plage number on the listed plages. The hope was to separate plages in terms of their chronology and identify each set of plages as pertaining to the same "underlying source of activity". This attempt at generating a data base failed primarily because of the incomplete listing of plages in the latter reference and uncertainty associated with the previous plage in a number of cases. Further, the supervening difficulty of 14-day gaps in the data makes this an unlikely base for meaningful prediction of flare activity.

Dr. Howard suggested that a complete and valid data base can be obtained in terms of the number of sunspots, as listed in "The Sunspot-Activity in the Years 1610-1960" by Prof. M. Waldmeier. In particular, he suggested that an attempt be made to predict the daily sunspot relative numbers over recent years as well as the Wolf number for the solar disk once every half-rotation and once every rotation. In this manner, the data would have less and less redundancy and should, therefore, become more and more difficult to predict. The prediction itself should reveal the obvious cycles and might indeed provide additional insight.

Data for the years 1957 and 1958 was analyzed in terms of the distribution of the daily number of sunspots and their first differences. These distributions were then partitioned into eight almost equally probable sections, these being 0, 131, 152, 169, 186, 205, 226, 252, and $+\infty$ for the daily number and $-\infty$, -24, -12, -6, 0, +6, -12, -24, and $+\infty$ for the first differences. A magnitude of the difference error-cost matrix was used with growth of experience permitted for each prediction. A penalty per state of 0.05 permitted only one state prediction machines to evolve. Decreasing the penalty to 0.01 resulted in more explicit descriptions of the apparent logic within the data base. In the next experiment 35.5% of the first 110 predictions were correct and 76.4% of these were correct within one symbol, these predictions being made by machines of increasing complexity ending in a 12 state predictor-machine. This experiment was repeated but with an increase in the length of the initial recall to 150 symbols. This resulted in the evolution of a 13 state machine in the first 40 predictions with 35% of these being correct, 87.5% being correct within one symbol. A repeat of this experiment with a change of the random number basis of mutation resulted in a similar 13 state predictor-machine and a score of 27.5% correct, 72.5% of these predictions were correct within one symbol. For the sake of comparison this same environment was given to the Markov prediction program. After 150 predictions the following scores were attained: 0.245, 0.231, 0.400, 0.433, 0.388, 0.390, 0.371, and 0.411. The first two of these scores were generated by presuming the environment to be a zeroth-order Markov provess. In the first of these each prediction corresponded to the symbol having highest marginal probability. The second score was generated in a similar manner but with prediction deferred in cases of equal probability of two or more symbols. The third pair of scores are generated under the presumption that the environment is a first-order Markov process, the second of these being the result of "conservative" prediction. The third pair were generated under the presumption of a second order Markov process while the last pair was generated by presuming that at each point in time the environment is of that order which has thus far exhibited the highest score over the zeroth-, first-, and second-order presumptions. Comparison of the scores appears to indicate that the evolutionary program had discovered some of the low order dependencies within the data but did not prove superior to consistently predicting on the basis of the best Markov presumption.

Turning attention to the first difference data base, with an initial recall of 40 symbols and a penalty of 0.01 complex machines of 26 states evolved but with little success, the percent correct score being 7.65% and 22.4% correct within one symbol. Such scores do not look too bad in comparison to those produced by the Markov program. After the first 150 predictions these scores were 0.164, 0.168, 0.145, 0.129, 0.151, 0.180, 0.108 and 0.100, these being given in the same order as above.

A new environment was then formed by placing the coded number of sunspots for the first day of each month in sequence. Using a penalty per state of 0.01 resulted in the evolution of a 22 state machine after the first 110 predictions. At this point the percent correct had reached 24.5% and the percent correct within one symbol had reached 47.2%. In comparison the Markov program produced the scores 0.164, 0.164, 0.278, 0.329, 0.272, 0.247 and 0.284 after the first 150 predictions. This environment was then converted to first differences yielding 20.9% correct prediction and 41.8% correct within one symbol. Recognizing that the first day of each month is a rather imperfect measure of the monthly activity level, attention was turned to predicting the sequence of monthly average number of sunspots. Here the evolutionary program produced an 18 state predictor-machine after the first 110 predictions and a score of 26.4% correct, 66.3% correct within one symbol. By way of comparison the Markov program produced 0.218, 0.218, 0.398, 0.375, 0.447, 0.432, 0.423, and 0.388 against the above stated presumptions. In general then these experiments tend to demonstrate that there are no significant short term dependencies within the sunspot activity as measured on a daily or monthly basis. These same experiments also indicate that there is no short term periodicity within these same data. At this point it is important to raise a caveat: the data as analyzed by the programs was encoded according to a rather reasonable but otherwise arbitrary rule...maximize the information content in the 8-symbol sequence of symbols (at least at the marginal level). If complex physical dependencies exist within the source of the data some other rule might well be required. Further, the encoding is based upon only the sample or real-world data upon which analysis was performed. Taking a broader picture might well afford a new and worthwhile bias. Then too, the data themselves were based upon a reasonable but otherwise arbitrary dictum which combines the observed number of spots and their area into a single measure.

Even with these difficulties clearly defined it appeared reasonable to proceed to develop a 64-symbol evolutionary program in the hope that with the added precision it might be possible to find some otherwise unexpected dependencies. Progress was made in this direction until some recent work by P. D. Jose was brought to our attention. In his paper entitled "Sun's Motion and Sunspots" Dr. Jose demonstrates a strong correspondence between solar activity as measured in terms of Wolf number and the gravitational field on the surface of the sun. For example, he states that the period determined from mechanical considerations is 178.77 years, $\sigma = 0.34$ while the period determined from sunspot activity is 178.55 years, $\sigma = 1.05$. Certainly this is a strong demonstration of gross correlation.

It would appear unreasonable to expect an evolutionary program to find the long term dependencies in view of the shortness of the sample with respect to this period. At the other extreme dependencies may exist within the data but these might best be found through a search for causal relations which relate various physical parameters. At this point in time such data is not available although a project is currently underway at AFCRC (under the direction of Dr. F. Ward) which is intended to draw together all available data in this regard. With such data in hand and with the kind of information which will be received from a solar probe it may be more meaningful to once again attempt to develop a more extensive evolutionary program for analysis of data.

Dr. Fred Ward of AFCRL indicated that his group is nearing completion of the first phase of their task...the compilation of all flare data for the years 1955 through 1964. These data will be ready in about a month in the form of a magnetic tape containing some 80 to 90 thousand flares by time, area, and importance. Every effort is being made to correct the areas reported for each flare but no effort is being made to correct the importance since these are defined in terms of the area. In Dr. Ward's opinion, only the corrected area data has any real validity. In the months which will follow, the data base will be increased to include plages, active regions, magnetic measurements, and sunspots. This more complete data base should be ready in about six months.

Conclusion

Evolutionary programming offers a versatile means for the prediction and analysis of time series. Initial effort under this contract was devoted to the preparation of an evolutionary program suitable for prediction of environments which are described in terms of up to eight different symbols. With this program debugged and documented experiments were conducted to evaluate its capability in terms of predicting environments of increasing difficulty. The first series of experiments concerned environments which consisted of an arbitrary repetitive 8-symbol length sequence disturbed by increasing levels of different kinds of noise. In fact, in one case the noise was permitted to become so severe as to completely obscure the repetitive signal except for its probabilistic shadow. But even in such a case significant prediction was accomplished and the predictor-machines identified the remaining shadow.

The next series of experiments were conducted against stochastic environments of increasing order of dependency. It soon became evident that although the implicit memory of finite-state machines can contribute to successful prediction of higher-ordered Markov processes, the limitation of explicit memory to the first conditional dependency served as a fundamental constraint. A predicting technique has been developed which provides extended memory to the evolving organism so that it may prove suitable for the prediction of any finite-ordered Markov process within the available size of the alphabet. More specifically, the precoding consists of identifying each fixed length overlapping sub-sequence of the incoming data with an unique symbol in a larger alphabet then performing conventional evolution in this new alphabet.

Having met with success in this regard attention was turned to the prediction of nonstationary environments. For the sake of having an available point of comparison environments were constructed which were deterministically generated but did appear to be nonstationary when examined in terms of their statistical properties. In situations wherein this nonstationarity consisted of a gradual trend the prediction achieved considerable success through growth of the recall. Since then further related experiments have been performed in which the environment proceeded in a stationary fashion until some point at which it grossly changed its properties to those of a new stationary process. For such environments it is well to limit the recall and at the same time extend the memory of the evolving automata. Much remains to be determined concerning these parameters. Beyond this point a number of experiments were conducted which demonstrated the capability of the evolutionary program for the prediction of multivariate environments.

With these results as sufficient justification, attention was turned to the prediction of real-world data. The magnitude and temporal order of solar flares were coded into an 8-symbol language and predicted with a score slightly higher than that which was accomplished by predicting the same data under the assumption of individual low-order Markov properties. Unfortunately, the large flares were so rare in the data base as to preclude their prediction with any significant confidence. Recognition of the need for greater specificity of time of occurrence dictated a fresh orientation in the data base. Obviously, partition of the data base by locale on the solar disk might increase the predictability of significant events through the reduction of irrelevant data within the data base. A review of the literature revealed a number of significant difficulties. First, half of the data concerning solar activity is missing (presently unobservable). This introduces further defects in the data in that plages which pass beyond the rim cannot always be identified with certainty as to their return. An attempt was made to separate plage and flare data in terms of life span. It was hoped that this procedure would permit constructing a data base comprised of flares or plages which were born while under observation (thus separating the physics of birth from that of later growth decay and burnout). Unfortunately, the data base was insufficient for accomplishing this. An attempt was made to examine correlative parameters such as type and area of sunspots, magnetic field strength, position, etc. But here again difficulty was encountered in finding an internally consistent data base.

This problem was reviewed with a number of specialists in the field of solar physics. At the suggestion of Dr. R. Howard of the Palomar - Mount Wilson Observatory, attention was focused upon sunspot data as published by Waldmeier. These data provided an adequate time sequence, were single dimensional, and are presumed to be reasonably accurate. The data, classified into 8-symbols in various ways, was predicted by the evolutionary program with results comparable to those obtained by assuming various low-orders of Markov dependency.

A number of experiments were performed in this regard, both on the evolutionary program and on a Markov prediction program. The results indicate that no low-order dependencies in the sequences of symbols which represented daily number, first difference of daily number, first day of successive months, average monthly number, and first difference of average monthly number of sunspots. The prediction capability of the evolutionary program appeared comparable to that of the Markov program. Although any cyclic properties within the data would be called out by the evolutionary prediction program, none was evidenced. Of course, the failure of these experiments to yield significant results may be due to use of an inappropriate encoding of the real-world data, inadequacy of the sample size, or errors which rest within the observed data as expressed in terms of Wolf number. Serious consideration was given to the possibility of enlarging the evolutionary program to a 64-symbol alphabet capability until recent work by P. D. Jose demonstrated a strong correlation between solar activity and the gravitational force on the surface of the sun. For example, according to Jose the period determined from mechanical considerations is 178.77 years ($\sigma = 0.34$) while the period determined from sunspot activity is 178.55 years ($\sigma = 1.05$). With such strong evidence in hand it is considered inappropriate to search for hidden statistical dependencies at a level of precision which promises to be well within the noise level of the encoding. Turning attention toward short term prediction would require a more adequate data base than is currently available. There is a strong possibility that a currently active project under the direction of Dr. F. Ward (AFCRC) will provide a firm data base for very short term prediction and analysis of solar activity. The significant gaps in current measurements will have to await the advent of solar probes.

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ADDENDUM

FORCASTING SOLAR ACTIVITY

By U. I. Vitinskii

"Prognozy Solnechnoi Aktivnosti." Glavnaia Astronomicheskaia Observatoriia, Izdatelstvo Akademii Nauk SSSR, Moskow, 1963

NASA Editor's Note:
This addendum consists of excerpts taken from a translation of a Russian work. Included is a detailed translation of Chapter 3, "Predictions of Mean Term Solar Activities," as well as the "Conclusions." The contents of this Russian document are outlined as a matter of general interest.

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Chapter 3. Predictions of Mean Term Solar Activities.

1. General Considerations

As already mentioned in the introduction predictions of mean term solar activities shall be meant to cover the prediction of monthly and quarterly solar indexes, within which we shall here limit ourselves to predicting sunspot numbers.

From Tables I and III in the Appendix it can be seen that the quarterly, and especially the monthly, Wolf numbers are very variable quantities. As a result, the task of predicting these numbers is significantly difficult and in the case of fluctuating solar activity the predictions are usually marked by considerable errors.

Unlike the annual Wolf number, which can be predicted several years in advance, the monthly numbers of sunspots (if their smoothed values are not taken into account) can be predicted only one month in advance with the present methods. The situation is somewhat better with respect to quarterly Wolf numbers. However, all methods for mean-time prediction yield sufficiently high errors so that the probability of these predictions is lower than the predictions of annual numbers of sunspots.

Since a number of methods for predicting quarterly and monthly Wolf numbers are based on the initial idea of the Mayot method we shall start with this very method, even though, strictly speaking, it was initially developed for monthly number of sunspots and, therefore, it should be considered somewhat later.

We note, that Mayot's method was subsequently expanded to the annual Wolf numbers but the probability of the given method, in this case, turned out to be so low that it scarcely deserves any mention.

2. The Mayot Method

In his paper Mayot (1947) starts out with the assumption that Wolf numbers for a number of years can be expressed in form of

$$W(t) = F(t) + E, \qquad (3.1)$$

where F(t) is the sum of trigonometic or exponential functions, and E a random quantity.

In this case the quantities W_i can be expressed by the formulas which take on the form

$$\left.\begin{array}{l} W_i = a_1W_{i-1} + a_2W_{i-2} + \ldots + a_iW_0 + \varepsilon_i, \\ W_{i+1} = a_1W_i + a_2W_{i-1} + \ldots + a_iW_1 + \varepsilon_{i+1}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ W_n = a_1W_{n-1} + a_2W_{n+2} + \ldots + a_iW_{n-i} + \varepsilon_n. \end{array}\right\} \qquad (3.2)$$

Here, the coefficients $a_1, a_2, \ldots, a_i$ are determined by the method of least square error provided the condition exists that $\sum_{k=i}^{n} \varepsilon_k^2$ is a minimum.

Originally, the Mayot method was worked out for smoothed monthly Wolf numbers which differ from the observed numbers in that their fluctuations are significantly less noticeable. For the known data this method gave a very small standard deviation of $\pm$ 1.9. However, when it is used for predictions the accuracy of the Mayot method decreases significantly.

Since the suggested Mayot formula for predicting smoothed monthly Wolf numbers appears to be erroneous (Vitinskii, 1956a) we shall not give it here. The correct formula will be given further on in conjunction with the corresponding method for predicting smoothed monthly numbers of sunspots.

Let us mention here one more circumstance. Mayot attempted to calculate the Wolf number for the same month and obtained a standard deviation of $\pm$ 9.2 for the numbers of sunspots for the months of January between the years of 1896 and 1944. However, here too, the error significantly increases when this is applied to predictions. Moreover, in this case, Mayot's basic premise is completely erroneous. The fact is, that the annual variation of Wolf numbers is far from manifest in all cycles, and at that, it is so weak that it could hardly serve as a guide.

As mentioned earlier, Moyot's method in the case of quarterly and monthly Wolf numbers results in considerable errors. But even though these numbers may deviate to either side by 20 to 25%, owing to the accidental variations of the coefficient k, as we shall see below, nevertheless these errors insure a sufficiently accurate prediction. Therefore, the application of the Mayot method to the mean term predictions appears to be sufficiently justified. As far as the prediction of monthly number of sunspots is concerned Mayot's method appears to be unique.

3. Method for Predicting Quarterly Wolf Numbers for the Following Quarter

For the prediction of Quarterly Wolf numbers (for the following quarter) U. I. Vitinskii (1956b, 1960a, 1961c) suggested the following three methods: The regression method, the Mayot method and the modified Mayot method. First let us look at the Mayot method. As already mentioned in the preceding paragraph, in order to apply this method it is first necessary to solve the system of equationsshown in (3.2). Such a system can be solved only if the coefficients of standard equations are sufficiently well developed. Therefore, it is expedient to use data which ends after the period of the maximum of the solar cycle or near to it, since especially during this period the Wolf numbers are sufficiently large and thus can ensure that the stated condition will be fulfilled. U. I. Vitinskii used the Zurich quarterly Wolf numbers for the years of 1944 to 1959. As a result of solving the systems of equations (3.2), the following relationship was obtained which can be used in predicting quarterly Wolf numbers for the following quarter:

$$W_5 = 0.92W_4 + 0.04W_3 + 0.25W_2 - 0.24W_1. \qquad (3.3)$$

The data for 1945 to 1959 obtained with the aid of the relation (3.3) gave a standard deviation of $\pm$ 24 for $\bar{W}_i$ = 106, that is, a relative standard deviation of $\pm$ 24%. It should be mentioned, that the greatest deviations of calculated quantities from the observed were associated with strong fluctuations of the Wolf numbers. In these cases the relative error was as high as 47%.

The modified Mayot method is also based on Mayot's idea, but it differs from this author's ordinary method in that it does not use the quarterly Wolf numbers themselves but rather the deviation of these numbers from an average curve. This method was applied with the goal to improve the development of coefficients of standard equations when using Zurich data for the years 1940 to 1955.

The average curve for Wolf numbers was obtained by the following methods. Let us assume that the average lengths of sunspots cycle is about 11 years as is apparent from observations. For such a cycle it is possible to construct an average curve of annual numbers of sunspots using the formula by Stewart and Panofski (1.10) with the coefficients a = +7.1832 and b = +1.2013 per Gleissberg (1951a). It will be expedient to start out here with the intensity of the cycle W_m = 100, that is F = 0.3473. The quantity θ is figured from the period of the minimum of the solar cycle.

If we assume that a and b are constant throughout all cycles, we can normalize such a curve for any concrete cycle on the basis of the ratio of its intensity to W_m = 100. The intensity of the cycle will be determined as the average of three annual Wolf numbers taken during the year of the maximum, the year after, and the year after that. The curve for the current cycle was constructed on the basis of predicted numbers with the aid of the method given in Chapter 2. Figure 10 shows examples of such curves for the cycles 17, 18 and 19.

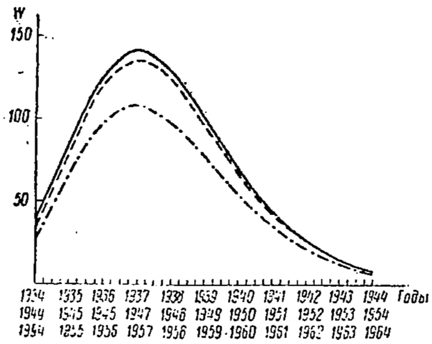

Рис. 10.

Now, by taking the smoothed quarterly Wolf numbers $\bar{W}_i$ from the average curves we can plot the difference between these and the observed numbers of sunspots W_i^0.

$$\Delta W_i = W_i^0 - W_i. \tag{3.4}$$

Solution of the system of equation of the form

$$\begin{aligned} \Delta W_i &= a_1 \Delta W_{i-1} + \\ &+ a_2 \Delta W_{i-2} + \ldots + \\ &+ a_i \Delta W_0 + \varepsilon_i \qquad (3.5) \end{aligned} \tag{3.5}$$

by the method of least square we can obtain then the desired relationship for the prediction of the quantity ΔW_i. Let us look at the solution of the system in the type (3.5) with 6 unknowns. The results which will also give the relationship

$$\Delta W_7 = 0.64 \Delta W_6 + 0.17 \Delta W_5 + 0.20 \Delta W_4 - 0.12 \Delta W_1. \tag{3.6}$$

The values ΔW_3 and ΔW_2 are omitted since they are negligibly small.

By knowing the corresponding quantity W_i, obtained from the average curve, we can obtain the desired quarterly Wolf number by the formula

$$W_7 = \bar{W}_7 + \Delta W_7. \tag{3.6a}$$

The quarterly Wolf numbers for the years 1941 to 1955 obtained with the equations (3.6) and (3.6a) gave a standard deviation of $\pm$ 20 for the mean value of the quarterly number $W_i = 61$, that is, a relative standard deviation of $\pm$ 33%.

The modified Mayot method, as seen from the above numbers, gives a relatively low accuracy. To a significant degree this is due to the condition that the given method appears to be dual. On the one hand it is guided by the Stewart-Panofski curve which is based on the very long-term predictions of the current cycle. On the other hand it includes the predictions of the disturbed part of ΔW_i. Since this is the case, it must be expected that the accuracy of the given method is lower than any of the other methods contained within it.

Speaking of the shortcomings of the modified Mayot method one must mention that it also has the weak point that it is not possible to predict even nearly accurately the quarterly numbers of sunspots during a period of strong fluctuations.

The method of regression suggested by A. I. Olem (1954) for predicting annual Wolf numbers may be applied with sufficient success, as demonstrated by the statistical survey of the Zurich material for 19 incomplete solar cycles, and also for the quarterly values of this index.

In order to apply the method of regression it is necessary to know fairly accurately the period of maximum and minimum of solar cycles. The methods of predicting these values will be discussed in Chapter 4, and therefore, we shall not dwell on them here. Let us only mention that we are entirely satisfied when we determine the periods of the extremes with an accuracy of one quarter.

Examination of the Zurich data for an increasing portion of the 19 solar cycles resulted in the following equations of regression as well as the corresponding coefficients r and the standard deviations σ:

$$\left.\begin{array}{llll}
W_{m+1} = 1.08W_m & + 4 & r = +0.55 & \sigma = \pm 4 \\
W_{m+2} = 1.36W_{m+1} & + 3. & r = +0.70 & \sigma = \pm 6 \\
W_{m+3} = 0.69W_{m+2} & + 3 & r = +0.81 & \sigma = \pm 4 \\
W_{m+4} = 1.08W_{m+3} & + 5 & r = +0.71 & \sigma = \pm 8 \\
W_{m+5} = 1.42W_{m+4} & & r = +0.89 & \sigma = \pm 9 \\
W_{m+6} = 1.20W_{m+5} & + 6 & r = +0.92 & \sigma = \pm 10 \\
W_{m+7} = 1.11W_{m+6} & + 7 & r = +0.90 & \sigma = \pm 12 \\
W_{m+8} = 1.13W_{m+7} & + 4 & r = +0.94 & \sigma = \pm 12 \\
W_{m+9} = 0.98W_{m+8} & + 12 & r = +0.91 & \sigma = \pm 15 \\
W_{m+10} = 1.17W_{m+9} & - 7 & r = +0.94 & \sigma = \pm 17 \\
W_{m+11} = 0.96W_{m+10} & + 11 & r = +0.95 & \sigma = \pm 17 \\
W_{m+12} = 1.26W_{m+11} & - 2 & r = +0.97 & \sigma = \pm 13 \\
W_{m+13} = 1.09W_{m+12} & - 3 & r = +0.97 & \sigma = \pm 10 \\
W_{m+14} = 1.18W_{m+13} & - 8 & r = +0.96 & \sigma = \pm 14 \\
W_{m+15} = 0.92W_{m+14} & + 9 & r = +0.90 & \sigma = \pm 11 \\
W_{m+16} = 1.40W_{m+15} & - 10 & r = +0.88 & \sigma = \pm 15 \\
W_{m+17} = 1.20W_{m+16} & + 1 & r = +0.97 & \sigma = \pm 9
\end{array}\right\} \qquad (3.7)$$

From equation (3.7) can be seen that predictions of quarterly Wolf numbers for the first four quarters cannot be considered sufficiently reliable. For all increasing portions $\sigma = \pm 12$ when $\overline{W}_i = 49$, i.e., the corresponding standard deviation is about $\pm$ 24%.

Similarly, for the decreasing portion were obtained the following equations of regression:

$$
\left.\begin{array}{lll}
W_{M+1} = 0.92W_{M} - 13 & r = +0.96 & \sigma = \pm 12 \\
W_{M+2} = 0.86W_{M+1} + 4 & r = +0.95 & \sigma = \pm 12 \\
W_{M+3} = 0.94W_{M+2} + 8 & r = +0.90 & \sigma = \pm 18 \\
W_{M+4} = 0.92W_{M+3} + 12 & r = +0.85 & \sigma = \pm 22 \\
W_{M+5} = 0.72W_{M+4} + 21 & r = +0.82 & \sigma = \pm 21 \\
W_{M+6} = 0.76W_{M+5} + 12 & r = +0.78 & \sigma = \pm 23 \\
W_{M+7} = 0.90W_{M+6} + 4 & r = +0.87 & \sigma = \pm 19 \\
W_{M+8} = 0.80W_{M+7} + 12 & r = +0.91 & \sigma = \pm 15 \\
W_{M+9} = 0.90W_{M+8} + 1 & r = +0.92 & \sigma = \pm 13 \\
W_{M+10} = 0.79W_{M+9} + 6 & r = +0.92 & \sigma = \pm 11 \\
W_{M+11} = 0.84W_{M+10} + 10 & r = +0.77 & \sigma = \pm 19 \\
W_{M+12} = 0.82W_{M+11} + 2 & r = +0.88 & \sigma = \pm 14 \\
W_{M+13} = 0.76W_{M+12} + 6 & r = +0.86 & \sigma = \pm 12 \\
W_{M+14} = 0.67W_{M+13} + 8 & r = +0.89 & \sigma = \pm 8 \\
W_{M+15} = 0.98W_{M+14} - 1 & r = +0.90 & \sigma = \pm 9 \\
W_{M+16} = W_{M+15} & r = +0.84 & \sigma = \pm 14 \\
W_{M+17} = 0.79W_{M+16} + 4 & r = +0.86 & \sigma = \pm 11 \\
W_{M+18} = 0.74W_{M+17} + 8 & r = +0.81 & \sigma = \pm 11 \\
W_{M+19} = 0.65W_{M+18} + 1 & r = +0.88 & \sigma = \pm 8 \\
W_{M+20} = 1.10W_{M+19} - 2 & r = +0.96 & \sigma = \pm 5 \\
W_{M+21} = 0.88W_{M+20} - 1 & r = +0.92 & \sigma = \pm 6 \\
W_{M+22} = 0.84W_{M+21} + 1 & r = +0.92 & \sigma = \pm 6 \\
W_{M+23} = 0.70W_{M+22} + 1 & r = +0.83 & \sigma = \pm 5 \\
W_{M+24} = 0.89W_{M+23} & r = +0.85 & \sigma = \pm 7 \\
W_{M+25} = 0.70W_{M+24} & r = +0.88 & \sigma = \pm 6 \\
W_{M+26} = 1.02W_{M+25} & r = +0.85 & \sigma = \pm 7
\end{array}\right\} \qquad (3.8)
$$

For the entire increasing portion $\sigma = \pm 14$ when $\overline{W}_i = 52$, that is the relative standard deviation was equal to $\pm 27\%$. If we compare the accuracy of all three methods, described above on the basis of known data we find that the method of regression and the Mayot method result in less errors than the modified Mayot method. However, if we take into account the characteristic of the Mayot method that by changing over from actual data to predicted data the quantities predicted by this method, become increasingly more erroneous, than we can assume that the most favorable method is the method of regression. Unfortunately, for solar cycles with increasing portions of more than 17 quarters or with decreasing portions of more than 26 quarters one can use only the ordinary and modified Mayot method for the corresponding quarters.

Let us mention yet a technical detail which is related to all the methods of predicting quarterly Wolf numbers for the following quarter. Since the prediction must be made at the end of the preceding quarter it is necessary to use the preliminary values of quarterly relative numbers of sunspots for 83 to 84 days (out of 90 to 91). In general this has little effect on the accuracy of the predictions except in the case of strong fluctuations of solar activity.

Finally, at the present time it seems possible to obtain a judgement on the probability of predicting quarterly Wolf numbers with the data already obtained and with the aid of the modified Mayot method. The examination of the material from the first quarter of 1956 to the third quarter of 1960 shows that the standard deviation for this period amounted to $\pm$ 27 when $\overline{W}_i = 162$, that is $(1 - \frac{\sigma}{\overline{W}_i})$ 100% = 84%. This is significantly greater than the value $(1 - \frac{\sigma}{\overline{W}})$ 100% obtained with known data (67%).

4. Method of Predicting Quarterly Wolf Numbers two Quarters in Advance

For many practical purposes, especially for certain problems in geophysics and radiophysics, it is very important to increase the short term predictions of quarterly Wolf numbers. A direct approach to solving this question by way of multiple or even dual predictions leads to considerable error and therefore, can hardly be considered applicable.

In the first step towards solving this, let us narrow down the task to working out a method of predicting quarterly numbers of sunspot for two quarters in advance. To this end, we should use the follwoing quantities: ordinary semi-annual Wolf numbers W_i'; semi-annual Wolf numbers obtained by shifting the half-year one quarter back, which subsequently will be called W_i''. quarterly Wolf numbers either observed W_i^0 or predicted W_i.

U. I. Vitinskii (1960g, 1961c) proposed two approaches for solving this problem.

The first approach consists of making preliminary computations of the ordinary and separate semi-annual Wolf numbers which, together with observed quarterly Wolf numbers will enable one to predict quarterly numbers of sunspots for two quarters in advance. For this the relationship can be used

$$\left.\begin{aligned} W_{\mathrm{I}} &= 2W''_{\mathrm{I}} - 2W'_{\mathrm{II}} + W^0_{\mathrm{III}}, \\ W_{\mathrm{II}} &= 2W'_{\mathrm{I}} - 2W''_{\mathrm{I}} + W^0_{\mathrm{IV}}, \\ W_{\mathrm{III}} &= 2W''_{\mathrm{II}} - 2W'_{\mathrm{I}} + W^0_{\mathrm{I}}, \\ W_{\mathrm{IV}} &= 2W'_{\mathrm{II}} - 2W''_{\mathrm{II}} + W^0_{\mathrm{II}}. \end{aligned}\right\} \tag{3.9}$$

Here, in the case of the quantities W_i' and W_i'' the indexes I and II designate respective half years and in the case of W_i and W_i^0 the indexes I to IV designate the quarters of the corresponding year.

The second approach consists in predicting ordinary and separate semi-annual Wolf numbers as well as quarterly Wolf numbers, whereby a combination of these will give a means for predicting quarterly numbers of sunspots with the aid of the relationship.

$$\left.\begin{aligned} W_{\mathrm{I}} &= 2W''_{\mathrm{I}} - W_{\mathrm{IV}}, \\ W_{\mathrm{II}} &= 2W'_{\mathrm{I}} - W_{\mathrm{I}} . \\ W_{\mathrm{III}} &= 2W''_{\mathrm{II}} - W_{\mathrm{II}}, \\ W_{\mathrm{IV}} &= 2W'_{\mathrm{II}} - W_{\mathrm{III}}. \end{aligned}\right\} \tag{3.10}$$

As mentioned earlier, for a similar type of prediction it is necessary to make preliminary computation of semi-annual Wolf numbers. To this end, two methods can be used: the method of regression, and the Mayot method.

On the basis of examined Zurich data of 19 incomplete cycles the following equations of regressions were obtained for the increasing and decreasing portions of the solar cycle as well as the corresponding values of r and σ for ordinary semi-annual Wolf numbers:

$$\left.\begin{array}{llll}
W'_{m+1} = 1.53W'_{m} + 3 & r = +0.64 & \sigma = \pm 5 \\
W'_{m+2} = 1.30W'_{m+1} + 6 & r = +0.59 & \sigma = \pm 13 \\
W'_{m+3} = 1.21W'_{m+2} + 12 & r = +0.81 & \sigma = \pm 14 \\
W'_{m+4} = 1.28W'_{m+3} + 9 & r = +0.89 & \sigma = \pm 16 \\
W'_{m+5} = 1.25W'_{m+4} + 7 & r = +0.95 & \sigma = \pm 14 \\
W'_{m+6} = 1.01W'_{m+5} + 14 & r = +0.92 & \sigma = \pm 17 \\
W'_{m+7} = 1.14W'_{m+6} + 2 & r = +0.96 & \sigma = \pm 14 \\
W'_{m+8} = 1.37W'_{m+7} - 8 & r = +0.85 & \sigma = \pm 18
\end{array}\right\} \qquad (3.11)$$

$$\left.\begin{array}{llll}
W'_{M+1} = 0.84W'_{M} - 7 & r = +0.93 & \sigma = \pm 12 \\
W'_{M+2} = 0.96W'_{M+1} + 1 & r = +0.94 & \sigma = \pm 15 \\
W'_{M+3} = 0.79W'_{M+2} + 11 & r = +0.84 & \sigma = \pm 12 \\
W'_{M+4} = 0.84W'_{M+3} + 2 & r = +0.96 & \sigma = \pm 10 \\
W'_{M+5} = 0.89W'_{M+4} - 6 & r = +0.95 & \sigma = \pm 9 \\
W'_{M+6} = 0.76W'_{M+5} + 5 & r = +0.92 & \sigma = \pm 9 \\
W'_{M+7} = 0.87W'_{M+6} & r = +0.89 & \sigma = \pm 10 \\
W'_{M+8} = 0.58W'_{M+7} + 8 & r = +0.80 & \sigma = \pm 6 \\
W'_{M+9} = 0.85W'_{M+8} + 1 & r = +0.73 & \sigma = \pm 12 \\
W'_{M+10} = 0.77W'_{M+9} & r = +0.89 & \sigma = \pm 8 \\
W'_{M+11} = 0.82W'_{M+10} & r = +0.91 & \sigma = \pm 6
\end{array}\right\} \qquad (3.12)$$

The known data have on the increasing portion a standard deviation of $\pm$ 14 when $\overline{W}'_i$ = 51, that is, a relative standard deviation of $\pm$ 27%. For the decreasing portion the standard deviation amounted to $\pm$ 10 when $\overline{W}'_i$ = 56, that is, the relative standard deviation equals $\pm$ 18%. Let us mention that according to the relationship (3.11), the prediction of W_i' for the first two half years of the increasing portion appears to be unreliable.

For separate semi-annual Wolf numbers, based on the same material, the following equations of regression were obtained for the increasing and decreasing portions of the solar cycle and the corresponding values of r and σ:

$$\left.\begin{aligned}
W''_{m+1} &= 1.33W''_{m} + 5 & r &= +0.58 & \sigma &= \pm 6\\
W''_{m+2} &= 1.72W''_{m+1} - 1 & r &= +0.81 & \sigma &= \pm 9\\
W''_{m+3} &= 1.70W''_{m+2} + 4 & r &= +0.93 & \sigma &= \pm 10\\
W''_{m+4} &= 1.32W''_{m+3} + 10 & r &= +0.94 & \sigma &= \pm 16\\
W''_{m+5} &= 1.20W''_{m+4} + 5 & r &= +0.96 & \sigma &= \pm 12\\
W''_{m+6} &= 1.17W''_{m+5} - 3 & r &= +0.95 & \sigma &= \pm 17\\
W''_{m+7} &= 1.05W''_{m+6} + 14 & r &= +0.95 & \sigma &= \pm 15\\
W''_{m+8} &= 1.20W''_{m+7} + 9 & r &= +0.84 & \sigma &= \pm 17\\
W''_{m+9} &= 0.98W''_{m+8} - 1 & r &= +0.80 & \sigma &= \pm 17
\end{aligned}\right\} \quad (3.13)$$

$$\left.\begin{aligned}
W''_{M+1} &= 0.82W''_{M} & r &= +0.96 & \sigma &= \pm 10\\
W''_{M+2} &= W''_{M+1} & r &= +0.93 & \sigma &= \pm 14\\
W''_{M+3} &= 0.77W''_{M+2} + 10 & r &= +0.92 & \sigma &= \pm 12\\
W''_{M+4} &= 1.12W''_{M+3} - 17 & r &= +0.92 & \sigma &= \pm 13\\
W''_{M+5} &= 0.67W''_{M+4} - 9 & r &= +0.88 & \sigma &= \pm 12\\
W''_{M+6} &= 0.96W''_{M+5} - 9 & r &= +0.83 & \sigma &= \pm 13\\
W''_{M+7} &= 0.58W''_{M+6} + 10 & r &= +0.94 & \sigma &= \pm 7\\
W''_{M+8} &= 0.98W''_{M+7} - 1 & r &= +0.84 & \sigma &= \pm 11\\
W''_{M+9} &= 0.62W''_{M+8} + 5 & r &= +0.80 & \sigma &= \pm 10\\
W''_{M+10} &= 0.77W''_{M+9} - 2 & r &= +0.91 & \sigma &= \pm 6\\
W''_{M+11} &= 0.75W''_{M+10} & r &= +0.94 & \sigma &= \pm 4\\
W''_{M+12} &= 0.88W''_{M+11} - 2 & r &= +0.94 & \sigma &= \pm 4
\end{aligned}\right\} \quad (3.14)$$

In this case the standard deviation for the increasing portion was $\pm$ 13 for $\overline{W}''_i$ = 51, that is, the relative standard deviations amounted to $\pm$ 25%. For the decreasing portion the standard deviation was equal to $\pm$ 11 for $\overline{W}''_i$ = 52 which corresponds to relative standard deviation of $\pm$ 21%. The prediction for the first separate half year of the increasing portion is unreliable as can be seen from the relationship (3.13).

In the application of the Mayot method we used ordinary and separate semi-annual Wolf numbers of the Zurich series for the year of 1935 to 1959. In both cases this material gave sufficiently well developed coefficients for the standard equations.

For the prediction of ordinary semi-annual numbers of sunspots the following relationship can be used:

$$W'_5 = 1.22W'_4 + 0.09W'_3 - 0.28W'_2 - 0.10W'_1. \tag{3.15}$$

The data obtained by this formula gave a standard deviation of $\pm$ 19 for $\overline{W}'_i$ = 82, that is, a relative standard deviation of about $\pm$ 23%.

For determining the separate semi-annual Wolf numbers with the aid of the Mayot method the following relationship was obtained.

$$W''_5 = 0.91W''_4 + 0.60W''_3 - 0.37W''_2 - 0.22W''_1. \tag{3.16}$$

In this case the standard deviation amounted to $\pm$ 21 for $\overline{W}''_i$ = 80 which amounted to a standard deviation of $\pm$ 26%.

From the above it can be seen that there are eight methods which can be used for predicting the quarterly Wolf numbers for two quarters in advance:

1) The method for regression for ordinary and separate semi-annual Wolf numbers by using the formulas (3.9), (3.11), (3.12), (3.13) and (3.14);

2) The Mayot method for ordinary and separate semi-annual Wolf numbers by using the formulas (3.9), (3.15), (3.16);

3) The method of regression for ordinary and separate semi-annual numbers and the modified Mayot method for quarterly Wolf numbers by using the formulas (3.10), (3.11), (3.12), (3.13), (3.14), (3.6) and (3.6a);

4) Method of regression for ordinary and separate semi-annual Wolf numbers and Mayot method for quarterly Wolf numbers by using the formulas (3.10), (3.11), (3.12), (3.13), (3.14) and (3.3);

5) Method of regression for ordinary and separate semi-annual Wolf numbers and the method of regression for quarterly Wolf numbers with the use of the formulas (3.10), (3.11), (3.12), (3.13), (3.14), (3.7) and (3.8);

6) The Mayot method for ordinary and separate semi-annual Wolf numbers and modified Mayot method for quarterly Wolf numbers with the use of the formulas (3.10), (3.15), (3.16), (3.6) and (3.6a);

7) The Mayot method for ordinary and separate semi-annual Wolf numbers and the Mayot method for quarterly Wolf numbers by using the formulas (3.10), (3.15), (3.16), (3.3);

8) The Mayot method for ordinary and separate semi-annual Wolf numbers and the method of regression for quarterly Wolf numbers with the use of the formulas (3.10), (3.15), (3.16), (3.7) and (3.8).

Each of these methods has its own merits and shortcomings. Above all it should be noted that methods 3, 4 and 5 give much smaller errors than the remaining methods. Acutally, the known data for 1945 to 1959 gave the following standard deviations σ and corresponding probability:

$$\left(1 - \frac{\sigma}{W_i}\right) 100\%$$

		$\left(1 - \frac{\sigma}{W_i}\right) 100\%$
1)	± 36	66%:
2)	± 44	55
3)	± 31	70
4)	± 26	75
5)	± 25	76
6)	± 36	63
7)	± 32	67
8)	± 41	59

The methods 1, 3, 4, and 5 are based on the method of regression which can only be used for specific intervals of time from the period of minimum and a period of maximum of the solar cycle (corresponding to 4.5 years and 6 years). This led to the fact that with the aid of the mentioned method the known data from the fourth quarter of 1953 to the fourth quarter of 1954 could not be used.

One other very important circumstance regarding the method of regression must be mentioned. In order to be able to use this method it is necessary to know beforehand the periods of the extremes of the solar cycle. The method developed to date, which will be described in Chapters 4 and 5, in each case allows one to determine this period within an accuracy of half year.

The method of regression is much less sensitive to strong fluctuations than the Mayot method, as can easily be seen by comparing the respective relationships.

The shortcomings of the Mayot method have already been mentioned earlier. Let us dwell on another shortcoming of the modified Mayot method which makes itself felt especially when combined with the Mayot method for semi-annual Wolf numbers. Generaly, the quarterly numbers of spots at the very beginning of the increasing portion of the solar cycle are higher when calculating by this method which causes the predicted quarterly numbers for two quarters in advance, obtained by the method no. 1, to be significantly lower. In this case as a very rough approximation a correction of +44 could be introduced in the first two years of the cycle. However, it should be mentioned that this is a very rough method even though it decreases the standard deviation from $\pm$ 38 to $\pm$ 26.

Since the Mayot method gives greater errors than the method of regression, naturally, more emphasis was put on methods 1, 3, 4 and 5 which are based on the method of regression. In order to increase the accuracy of quarterly Wolf numbers for two quarters in advance it is necessary to take the average values obtained with various method, whereby quantities calculated by methods 1, 3, 4 and 5 will be assigned category 2 and the remaining quantities assigned category 1. This approach, as was illustrated by the known data for the period of 1945 to 1959, significantly decreased the errors. The standard deviations of the predicted quarterly Wolf numbers, averaged by the stated method, was $\pm$ 25 when $\overline{W}_i = 98$, which means that the probability of the known data was equal to 75%. This figure was entirely adequate if one considers that with the methods of predicting quarterly Wolf numbers for the following quarter gave results having practically the same probability. As far as the basic shortcomings are concerned, the large errors of predictions made during the periods of strong fluctuations also applies in this case.

5. Method of Predicting Smoothed Monthly Wolf Numbers

As mentioned already in Section 2 of this Chapter the prediction of smoothed monthly Wolf number was first developed by Mayot (1947). Based on the same material as that used by Mayot (1931 to 1944) U. I. Vitinskii obtained in place of Mayot's erroneous formula the following relationship for prediction of the average monthly Wolf numbers, W_i, for the following month:

$$\overline{W}_5 = 0.99\overline{W}_4 + 1.22\overline{W}_3 - 1.70\overline{W}_2 + 0.49\overline{W}_1. \tag{3.17}$$

From the known data for these years a $\pm$ 1.9 standard deviation was obtained with the aid of equation (3.17). Such a small error makes this method very attractive. However, owing to the characteristics of calculating smoothed numbers of spots we are immediately faced with a great difficulty.

We know that the W_i numbers are generally determined with the aid of equation (2) of the Introduction. As can be seen from this equation it is not possible to find smoothed monthly Wolf numbers more than six months in advance to at any given moment. If all these values are predicted successively by the Mayot method then in the final result the errors will increase to such an extent that the advantages (with respect to its accuracy) offered by this method is practically reduced to zero. In order to bypass this difficulty U. I. Vitinskii (1956c) suggested use of the relationship between the observed and smoothed monthly Wolf numbers. The relationship is characterized by a coefficient of correlation of r = +0.93 and the equation of regression:

$$\overline{W}_i = 0.98W_i + 2. \tag{3.18}$$

Undoubtedly, the given approach decreases the accuracy when predicting with the Mayot method, especially in case of solar activity fluctuations where the smoothed numbers of spots found by the relationship (3.18) may be noticeably higher. Nevertheless, in this case the errors are smaller than when using the other method suggested by Mayot for Wolf numbers for same-name-months.

Let us mention yet another technical detail. In order to make predictions with the stated method, it is entirely sufficient to use preliminary monthly numbers of spots from 23 to 27 days (out of 30 to 31 days).

An examination of the data for the period from January 1956 to October 1959 showed that the application of the Mayot method for predicting smoothed monthly Wolf numbers for the following month gave a standard deviation of $\pm$ 27 at an average value of W_i = 168, which corresponds to a mean probability of 84%.

We have already mentioned that the application of equation (3.18) can introduce false fluctuations into the prediction of smoothed numbers of spots. This can be seen clearly from Figure 11 in which the solid-line curve represents the plot of smoothed monthly Wolf numbers for 1957 to 1958, computed on the basis of observed values, and the dotted-line curve shows the plot of numbers predicted with the Mayot method.

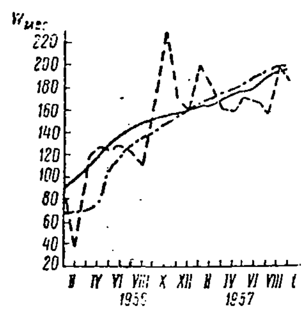

Fig. 11

In order to significantly counteract this shortcoming and, what is just as important, to get even shorter term predictions of smoothed monthly Wolf numbers, U. I. Vitinskii proposed a regression interpolation method which has the pleasant distinction of being simple and sufficiently accurate. Here is the description of this method.

In the previous paragraph we described a method of regression for ordinary and individual semi-annual Wolf numbers. The semi-annual numbers of spots can be considered as peculiar smoothed quantities insofar as they represent the average value of 6 monthly values of solar indexes we have used. Therefore these numbers can be applied to predicting smoothed monthly Wolf numbers. By knowing the semi-annual Wolf numbers for the first half of the given year (and relating it to April) and predicting it with the aid of the method of regression for the second half year (and relating it to October) it is possible, by way of interpolation to obtain predicted smoothed Wolf numbers for July, August, September and October of that year. Similarly, with separately predicted and observed semi-annual relative numbers of spots we obtain predicted smoothed Wolf numbers for October, November, December and January or for April, May, June, and July. With ordinary semi-annual numbers it is possible to obtain the values of the index examined by us for January, February, March and April.

Thus, the application of the regression interpolation method permits us to predict smoothed monthly Wolf numbers, even under least favorable cases, (in February, May, August, and November) for two months in advance and sometimes even for four months in advance (in March, June, September, and December). In this case the curve of the predicted smoothed monthly numbers of spots is much flatter than when using the Mayot method. This can be seen in Figure 11, where the numbers computed with the aid of the regression interpolation method are shown with the dot-dash line.

The known data for the period of January 1956 to October 1959 obtained by this method gave a standard deviation of $\pm$ 12 at a mean value of $\overline{W}_1 = 168$, that is a mean probability of 93%. In addition, if we take into account the fact that the regression-interpolation method permits us to make even significantly shorter short-term predictions of smoothed monthly Wolf numbers it becomes obvious that this method is more desirable than the Mayot method.

Let us discuss the method of predicting smoothed monthly Wolf numbers to the end of the current solar cycle worked out by Herrink (1958, 1959). This method is based on Anderson's (1954) assumption that there exists a 169-year cycle of solar activity. The agreement with the Wolf numbers for the period of 1749 to 1785 and 1918 to 1954 seems to be sufficiently precise.

Starting out with this, Herrink compared smoothed monthly numbers of spots at the beginning of the increasing portion of the 4-th and the 19-th solar cycle whereby July 1784 was taken to be the beginning of the fourth cycle and April 1954 the beginning of the 19-th cycle. He obtained the following equation of regression:

$$W_{19} = 1.488 W_4 - 12.5. \qquad (3.19)$$

Here the indexes indicate the number of the 11-year solar cycle.

A prediction based on this formula gives a standard deviation equal only to $\pm$ 1.6.

Striving for even more accurate predictions, Herrink used the data from April 1954 to October 1958 and obtained a new equation of regression:

$$W_{19} = 1.527 W_4 - 13.4. \qquad (3.20)$$

This equation differs very little from Equation (3.19). Equation (3.20) can be used for predicting smoothed monthly Wolf numbers to the end of the 19-th cycle.

In Table 10 are given the smoothed monthly Wolf number predicted for 1957 to 1958 by equation (3.19)(p) and for 1959 to 1969 by equation (3.20). For the purpose of comparison the observed figures (n) for 1957 to 1958 have also been given here.

Table 10. Predicted Smoothed Monthly Wolf Numbers for the 19-th cycle (per Herrink).

Таблица 10

Прогноз сглаженных месячных чисел Вольфа на 19-й цикл (по Хэррннку)

Месяц	1957		1958		1959	1960	1961	1962	1963	1964	1965	1966	1967
	п.	н.	п.	н.									
I	172	170	196	198	164	137	93	81	62	46	29	15	0
II	175	172	189	199	163	136	91	80	61	49	28	14	0
III	182	177	187	203	168	129	89	79	59	50	23	11	0
IV	185	183	180	198	165	122	87	78	58	47	19	11	0
V	184	187	179	191	167	117	86	77	57	46	19	9	0
VI	188	189	178	189	163	111	86	76	55	43	16	6	0
VII	191	191	176	187	160	108	87	75	51	40	19	5	0
VIII	189	190	177	182	157	106	85	73	52	41	19	2	0
IX	191	194	178	183	153	103	83	71	50	36	16	1	0
X	196	194	174	181	147	100	81	70	49	33	17	1	0
XI	197	197	169	181	143	97	80	68	48	31	16	0	0
XII	197	197	167	180	140	95	82	64	46	28	15	0	0

From November 1966 Equation (3.20) gives negative values which Herrink substituted by zeros.

With all its advantages the Herrink method has two significant shortcomings:

1. It is based on the existence of the 169-year cycle but it does not consider the established 88 to 90-year cycle sufficiently reliable. Owing to this, it is not sufficiently flexible and, strictly speaking, we should use it only for the 19-th cycle, and that not for the entire cycle.
2. Herrink's assumption that the two decreasing portions of the 4-th and the 19-th cycle are equal is obviously arbitrary. According to many papers the period of the minimum of the 20-th cycle is expected to occur not later than 1966. According to A. I. Olen (1960) it should fall in 1965.2. Therefore, the quoted values for 1964, 1965 are obviously too high.

Nevertheless, with slight modification the Herrink method can be used both for the prediction of smoothed monthly Wolf number as well as for predictions of quarterly Wolf numbers, not only for the 19-th solar cycle but also for any other cycle of the solar activity, especially for the extremely high intensity cycles.

6. Method of predicting Observed Monthly Wolf Numbers

As already mentioned in the Introduction the observed monthly Wolf numbers are highly fluctuating quantities. Therefore, the prediction of monthly numbers of spots, even one month in advance, represents a more complicated problem than long-term prediction of solar activity. This situation makes it permissible to consider predictions of these quantities with errors up to 25% entirely satisfactory because monthly Wolf numbers cannot be determined within these limits.

The only method by which monthly Wolf numbers for one month in advance can be predicted seems to be the Mayot method. By using the Zurich data for 1951 to 1956 U. I. Vitinskii (1960a) obtained for the prediction of monthly relative numbers of spots the following equation:

$$W_5 = 0.81W_4 - 0.14W_3 + 0.51W_2 - 0.19W_1. \tag{3.21}$$

Applying this formula to the data of 1944 to 1956, gave a standard deviation of $\pm$ 22, or a relative standard deviation of 27%.

Next, in order to increase the confidence level of the Mayot method application, an equation was worked out on the basis of Zurich data for 1954 to 1958 which more accurately characterizes the correct cycle:

$$W_5 = 1.32W_4 - 0.61W_3 + 0.82W_2 - 0.52W_1. \tag{3.22}$$

This equation gave for the known data a standard deviation of $\pm$ 22 for $\overline{W}_i$ = 123, which corresponds to a relative standard deviation of 18%.

The Mayot method gives the greatest errors during the period of strong fluctuations of solar activity. Therefore, in order to increase the accuracy of the predicted monthly Wolf numbers it is necessary to use certain artificial devices. Above all, one should note that at the present time we cannot predict even nearly reliably the beginning of strong fluctuations, not even within a quarter. Therefore, the approaches described below are mainly directed toward foreseeing the duration of fluctuations.

The examination of statistical material shows that, with the exception of rare cases, after a sharp increase the solar activity begins to fall off in the month following this increase. Therefore, in a very rough approximation these increases can be disregarded and, instead, the values taken from curves reflecting the general trend of increasing or decreasing activity in a given cycle can be used for predictions.

Second order approximations can be used for examining the development of long-lived groups of sunspots since fluctuations of solar activity are often caused by such groups. The basic characteristics of the development of long-lived groups of sunspots have been already described in paragraph 5 of Chapter 1. The application of these properties enables one to make purely quantitive judgements on the rate of decrease of the solar activity for the next revolution of the sun.

Finally, one more approximation can be performed. The active longitudes contribute significantly to the fluctuations of solar activity. The study of processes taking place at the active longitudes showed that they have a characteristic rhythm of an average period of 4 to 5 revolutions of the sun (Vitinskii, Rubashev, 1957). Considering that at any given time - interval, as a rule, one of the active longitudes diminishes, it will be possible to know with some degree of certainty when the next fluctuation of solar activity might be expected. However, it is necessary to make one important reservation. In the cited reference the active longitudes were computed with indexes of areas of sunspot groups which sometimes behave differently than the Wolf numbers. Therefore, the given results are not sufficiently reliable to be used for prediction if we deal with Wolf numbers and not with areas.

Finally, at the present time there seems to be available a method by which the probability of predicted monthly Wolf numbers for the following month, obtained with the aid of the Mayot method, can be judged. The examination of data for the period from July 1957 to September 1960 shows that the standard deviation for this period amounts to $\pm$ 28 for $\overline{W}_i = 168$, that is $(1 - \frac{\sigma}{\overline{W}_i})\ 100\% = 83\%$. This is a slightly higher probability than the one obtained with known data while from the characteristics of the Mayot method for smoothed Wolf number the opposite could be expected.

Let us also mention that at the present time there does not exist a possibility of obtaining a method which would permit predicting observed Wolf numbers even two months in advance.

7. Analogy Method for Predicting Quarterly Wolf Numbers

A small modification of the Herrink method, described in Section 5 of this chapter, makes it possible to predict quarterly Wolf numbers for the whole decreasing portion of the current cycle. By virtue of earlier indicated shortcomings of this method we shall be dealing with a slightly different initial condition. We shall base this on the two following facts:

1. According to Xanthakis (1959) one of the most important characteristics of the solar cycle appears to be the length of its increasing portion and therefore, we shall start out first with this characteristic;
2. In most cases the length of the decreasing portion of similar cycles differs from the length of this portion of the cycle under study. Therefore, its length can be estimated only with the help of existing methods for predicting the period of minimum of solar cycles.

Since the method suggested by U. I. Vitinskii (1960d, 1961c) is primarily based on the choice of analogy cycle for a given cycle we shall subsequently refer to it as the method of analogys.

The selection of similar cycles will be based on two criteria: Equal (or nearly equal) length of the increasing portion of the cycle under study or any given cycle and the closest relationship of the quarterly Wolf numbers on the increasing portion of both cycles, this expressed in terms of the coefficient of correlation. It must be mentioned here that the allowable difference cannot exceed one quarter in the ratio of the period of growth of the cycle under study and any given cycle (during the period of increase). Even if all obtained coefficients of correlations are high the cycle with the highest coefficient r is selected. In the cases when more than one cycle has the same coefficient of correlation all of the cycles are used.

The application of this method and the enunciation of the initial Herrink principles turn out to be entirely satisfactory. Actually, if we start out with the assumption that a 169-year solar cycle exists then the 17th and 18th 11-year cycles are supposed to be similar to the 2nd and 3rd cycles. However, as shown by U. I. Vitinskii the 17th cycle is similar to the 10th cycle and the 18th cycle is similar to the 11th cycle. And what is more, on the same basis the 4th and 13th cycle can be consdered to be similar to the 19th cycle.

In order to predict quarterly Wolf numbers on the decreasing portion toward the end of the cycle, it is necessary to make a comparison of the numbers on the increasing portion of the given cycle and those of the similar cycle in order to develop equations of regression similar to those developed by Herrink for the smoothed monthly numbers of spots. The known data for the decreasing portion of the 17th and 18th cycle, worked out with the aid of equations of regressions obtained for these cycles, gave a probability of approximately 63 and 72%. Such a difference in probability can be explained by the fact that weaker cycles fluctuate more whereby the period of strong fluctuations of the different cycles seldom coincide within an accuracy of one quarter.

On the basis of U. I. Vitinskii's method of analogy the following equation of regression was obtained for the 19th cycle:

$$W_{19} = 1.49W_4 + 2, \tag{3.23}$$

$$W_{19} = 2.41W_{13} + 2. \tag{3.24}$$

Here the indices designate the number of the cycle.

On the basis of known data for the period from the first quarter 1958 to third quarter 1960 the application of formula (3.23) gave a relative standard deviation of 17% and the application of equation (3.24) gave 16%. Insofar as both similar cycles used have approximately the same characteristics, we took the average values predicted on the basis of these. This also seems advantageous because the fourth cycle distinguished itself by an abnormally long decreasing portion (contrast of the 13th cycle) and therefore, would cause an increase in the values predicted for the last years of the current cycle. The average of the known data for the period of the first quarter 1958 to the third quarter 1960 resulted in a relative standard deviation of 15%.

Our examination showed that with the analogy method it is possible to predict quarterly Wolf numbers only for the decreasing portion. Since the decreasing portion has many different characteristics than the increasing portion it might be useful from the behavior of the decreasing to improve the accuracy of predicted numbers by bringing in additional data and recalculating the equation of regression.

If we bring in the data for the decreasing portion of the 19th cycle through the 3rd quarter of 1960, we obtain the following equation of regression:

$$W'_{19} = 1.39W'_{4} + 3, \tag{3.25}$$

$$W'_{19} = 2.37W'_{13} - 9. \tag{3.26}$$

The difference between these equations and equation (3.23), (3.24) is insignificant, but it is advantageous to use them since they reflect the trend of the decrease in solar activity in the current cycle.

Table 11 gives predicted quarterly Wolf numbers, which turn out to be the averages of quantities obtained with equation (3.25) and (3.26) for the period of 4th quarter 1960 to 1st quarter 1965. The period of minimum of the 20th cycle turns out to be 1965.2, in agreement with A. I. Olem (1960). This data which was computed in a similar manner but with the aid of equation (3.23) and (3.24), gives the quarterly numbers of spots for the period of first quarter 1958 to third quarter 1960 (p) and their deviations from observed Zurich quarterly numbers (p-n).

Table 11

Predicted quarterly Wolf numbers from 1958-1965 (per Vitinskii)

Quarter	1958		1959		1960		1961	1962	1963	1964	1965
	p	p-n	p	p-n	p	p-n					
I	204	+18	168	-14	157	+42	88	62	67	38	21
II	183	+2	170	-11	127	+1	86	87	52	38	
III	220	+22	171	-27	108	-22	64	60	37	35	
IV	190	+16	162	+42	104		81	66	39	40	

It should be noted that the tabulated data for 1964 to 1965 obviously appear to be higher, since even the length of the decreasing portion of the 13th cycle is greater than the predicted length of decreasing portion of the 19th cycle.

CONCLUSIONS

Having reviewed the basic empirical-statistical methods for long-term predictions of the Wolf numbers we come to the conclusion that the reliability of the results obtained is very far from that desired.

What are the possible roads along which solar activity predictions could be developed in the future? First of all, in order to solve this complicated problem it is necessary to have a complex approach. Even the most perfect theory on solar activity, if we can imagine that one could be developed in the future, will not be able to give completely reliable results. At the present time there is not even a likeness nor even a sufficiently developed outline of such a theory. Therefore, the first step must be to seek a way by which good results would be achieved even if unrelated theories on the physics of the sun would be applied.

The methods stated here have practically nothing to do with the morphology of solar activity. Nevertheless, the morphological approach to the problem of predicting solar activity could in our opinion give some insight. It will be particularly important in developing the method for short term predictions. The morphological method calls for examining the development of the centers of activity that is, all the layers of the solar atmosphere from top to bottom. Thanks to this, it will be possible to judge the future development of the activity responsible for sunspot formation by specific advanced changes in other layers of the solar atmosphere. In addition, by observing the radio emission of the sun in the centimeter band it will be possible to get a glance of the invisible solar hemisphere one or two days before the appearance of a group of sun spots on the eastern limb of the visible hemisphere (Molchanov, 1959; Ikhsanova, 1960). Consequently, the application of this data will give one of the ways by which a short term solar activity prediction could be made.

The morphological approach so far has been used little for monthly predictions. If applied at all, it was used as sort of a supplement to the statistical method. In this sense, it would be very interesting to develop the morphological method for monthly predictions to such a degree that it would be equal with the statistical method of Mayot, now in existence.

At the present time the only means for predicting solar activity (discounting the area of spots) seems to be the Wolf numbers, but if we are not concerned with extremely long term predictions, it would be very valuable to develop a method of predictions in which different solar activity indices would be used. It is possible that parallel predictions of different solar indices could be used to check the reliability, even if not of all then at least of many, of the predicted quantities.

Finally, the most important thing which would help predicting would be the development of a complete theory of solar activity. It could serve mainly as basis for developing theoretical method for predicting solar indices. Even if the first attempt made by B. M. Rubashev in 1954 did not yield sufficiently good results, the very formulation of this type of problem is in itself very valuable. Let us hope that in the not too distant future scientists will unlock the basic mysteries of the sun: what causes solar activities and what are their mechanisms? Not too long ago the hydrodynamic theory of solar activity by Bjerknes (1926) seemed entirely reliable. Next appeared the Al'fev theory (1952) which just about completely repudiated the Bierknes structure and substituted in its place the magnetohydrodynamic wave. At the present time we are witnessing the synthesis of these two directions which is being reverberated in the rapid development of magnetodydrodynamics. This is not the place to discuss the many questions which have been posed or even been solved by this young branch of physics and astrophysics. Let us only mention that at the present there is hardly a doubt that a fullfledged theory of solar activity could be developed without taking into account magnetic phenomena.

On the other hand, many scientist devote a lot of attention to studying the characteristics of the differential rotation of the sun and its connections with magnetic solar energy.

In short, these are the basic ways by which problems of predicting solar activities could be solved. Briefly stated, any studies related to the activities on the Sun, regardless of the direction they take, will in one way or other help to solve this most important question.

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